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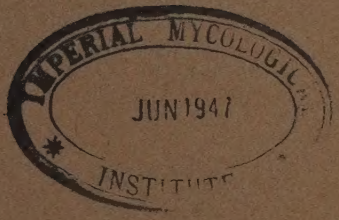

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MATHÉMATIQUES

LES SINGULARITÉS LOCALES DES POINTS ORDINAIRES D'UN SYSTÈME D'ÉQUATIONS DIFFÉRENTIELLES

Par E. BARBACHINE

(Présenté par A. N. Kolmogoroff, de l'Académie, le 12. VII. 1943)

1. Soit

$$\frac{y_i}{dx} = f_i(x, y_1, y_2, \ldots, y_n) \qquad (1)$$

un système d'équations différentielles où les fonctions f_i sont continues par rapport à tous les arguments dans un certain domaine Γ de l'espace à $n+1$ dimensions.

On peut considérer les solutions du système (1) comme un système de courbes situées dans l'espace à $n+1$ dimensions.

Soit $x = x_0$ un hyperplan à n dimensions qui coupe Γ; et soit $\overline{G}$ un certain domaine fermé situé dans l'intersection de Γ et de cet hyperplan. Alors en vertu des théorèmes généraux sur l'existence des solutions on peut toujours trouver des constantes positives a et b telles qu'il sort de chaque point de $\overline{G}$ une courbe ou bien une infinité de courbes intégrales dont chacune est déterminée pour les valeurs de x entre $x_0 - a = a'$ et $x_0 + b = b'$. L'ensemble des points situés sur les courbes intégrales issues des points de $\overline{G}$ et considérés pour les x tels que $a' \leqslant x \leqslant b'$, forme un ensemble fermé que nous désignerons par $\overline{A}$ et qui sera l'objet de nos considérations dans tout ce qui suit.

Considérons un hyperplan $x = x_0$ à $n+1$ dimensions et désignons par $G^{(n)}_{x_0}$ son intersection avec A.

Si l'on suppose que par chaque point p de $G^{(n)}_{x_0}$, $a'' \leqslant x_0 \leqslant b''$ (a'' et b'' étant des constantes), il ne passe qu'une seule courbe intégrale, alors la structure topologique de la famille des courbes intégrales remplissant un certain voisinage à $n+1$ dimensions du point p est complètement caractérisée par ce fait bien connu:

L'ensemble des courbes intégrales remplissant le voisinage d'un point ordinaire du système (1) est tel qu'on peut établir une correspondance biunivoque réciproque et continue dans les deux sens entre cet ensemble et un système de segments parallèles.

Si la condition d'unicité n'est pas remplie, la situation devient beaucoup plus compliquée: on peut observer une infinité de types topologiques différents dans la situation des courbes intégrales qui remplissent le voisinage des points ordinaires.

Pour le cas d'une seule équation $\frac{dy}{dx} = f(x, y)$ ce problème a été étudié par Brouwer, Charpantier, van-Kampen (1).

Dans la présente Note; en suivant les définitions proposées par V. Niemytzki, nous divisons les points ordinaires en deux classes: les points de continuité et de discontinuité, et nous montrons que les points de continuité présentent au point de vue descriptif la masse fondamentale parmi l'ensemble des points de A. Quant aux points de discontinuité, dans tous les exemples connus de systèmes où la condition d'unicité tombe en défaut il y a toujours des points de discontinuité, mais nous n'avons pas réussi à démontrer que cette circonstance a toujours lieu.

Passons aux définitions.

2. L'ensemble des courbes intégrales issues d'un certain point p et menées dans la direction positive sera nommé faisceau intégral et désigné par $H(p)$; d'une manière analogue on peut parler d'un faisceau intégral issu d'un certain ensemble E et le désigner par (E).

L'intersection du hyperplan $x=x_0$ et du faisceau intégral $H(p)$ ou $H(E)$ sera nommée section et désignée par S_{p,x_0} ou respectivement S_{E,x_0}. Considérons un certain ensemble E contenant p; si l'intersection de $H(p)$ avec E est un arc simple, le point p sera nommé point d'unicité par rapport à l'ensemble E. Considérons maintenant un ε-voisinage sphérique du point p, soit $U_\varepsilon(p)$, et un point $p' \subset U_\varepsilon(p) \cdot E$; considérons encore

$$\sup_{p' \subset U_\varepsilon(p) \cdot E} \alpha(H(p)E;\ H(p')E),$$

où $\alpha(A, B)$ est l'écart des deux ensembles A et B suivant la terminologie de M. Hausdorff (2). Il est évident que quand ε décroît, cette quantité ne peut que décroître, donc il existe

$$\alpha(p) = \lim_{\varepsilon \to 0} \{ \sup_{p' \subset H_\varepsilon(p) \cdot E} \alpha(H(p)E;\ H(p')E) \}.$$

Définition. *Un point $p \subset A$ est nommé point de continuité par rapport à l'ensemble E, si $\alpha(p)=0$, et il est nommé point de discontinuité, si $\alpha(p)>0$.*

Si l'on prend pour E la partie de A qui est située à droite du hyperplan $x=x_0$, nous parlerons d'un *point de continuité* ou bien d'un *point d'unicité* en A.

Définition. *Un point p situé sur le hyperplan $x=x_0$ sera nommé «point de continuité locale», s'il existe un nombre ε tel que ce point est un point de continuité par rapport à un voisinage cylindrique $Z(p)$ du point p dont le diamètre est égal à ε et qui est situé entre les hyperplans $x=x_0$, $x=x_0+h$, $h<\varepsilon$ et entièrement continu dans A.*

On donne une définition analogue pour un point «d'unicité locale». Il est évident que les points de continuité et les points d'unicité dans A sont en même temps points de continuité locale et d'unicité locale, mais la réciproque n'est pas vraie. Dans la terminologie habituelle on dit que l'unicité tombe en défaut en un certain point, si «l'unicité locale» tombe en défaut.

3. Pour démontrer les théorèmes qui suivent nous aurons besoin du corollaire suivant des théorèmes de Kamke (3).

Lemme. *Soit $A_1A_2 \ldots A_n \ldots$ une suite d'ensembles fermés situés sur le hyperplan $R: x=x_0$, continus les uns dans les autres et dont les diamètres tendent vers zéro pour $n \to \infty$; supposons que toutes les courbes intégrales issues de tous les points de chaque A_n peuvent être prolongées de $x=x_0$ jusqu'à $x=x_0+h$, et soit enfin p un point appartenant à tous les A_n. Alors pour chaque $\varepsilon>0$ il existe un N tel que pour chaque $n>N$ et pour $x_0 \leqslant x \leqslant x_0+h$ on a*

$$\alpha(S_{A_n,x};\ S_{p,x}) < \varepsilon.$$

Théorème 1. *L'ensemble des points d'unicité en A (respectivement, des points d'unicité locale) situés sur chaque hyperplan $R: x+x_0$ est contenu dan*

l'ensemble des points de continuité en A (respectivement, de continuité locale).

Démontrons ce théorème. Soit $p \subset R$ un point d'unicité en A et ε—un nombre positif arbitraire. En vertu du lemme, ε étant donné, on peut trouver un $\delta > 0$ tel que pour la sphère $U_\delta(p)$ de rayon δ et de centre p où a pour $x_0 \leqslant x \leqslant b'$ l'inégalité

$$\alpha\,(S_{U_\delta(p)\cdot R,x};\ S_{p,x}) < \varepsilon.$$

Cela signifie que si l'on décrit autour de chaque point une sphère $S_{p,x}$ de rayon ε, alors $S_{U_\delta(p)\cdot R,x}$ sera contenue entièrement à l'intérieur de cette sphère, donc l'ensemble $S_{p',x}$, où $p' \subset U_\delta(p) \cdot R$, sera aussi à l'intérieur de la sphère indiquée, et comme d'ailleurs l'ensemble $S_{p,x}$ se réduit à un seul point, on a $\alpha\,(S_{p',x_0}, S_{p,x}) \leqslant \varepsilon$ pour chaque x pour $x_0 \leqslant x \leqslant b'$, c'est-à-dire

$$\alpha\,(H(p'),\ H(p)) \leqslant \varepsilon$$

pour tous les points p' appartenant à $U_\delta(p)$. Cela signifie que $\alpha(p) < \varepsilon$, où ε est un nombre arbitraire. Il en résulte que $\alpha(p) = 0$. La démonstration pour les points d'unicité locale reste évidemment la même.

Théorème 2. *L'ensemble des points de continuité en A est sur chaque hyperplan $R: x = x_0$ un ensemble du type G_δ et de seconde catégorie (suivant la terminologie de Baire).*

Pour le démontrer enfermons A en un système fini $\sum_n$ de sphères fermées S dont le diamètre ne surpasse pas $\frac{1}{n}$. Soit $\sum'$ un ensemble arbitraire de sphères prises dans $\sum_n$. Désignons par $F_{n,\Sigma'}$ l'ensemble des points de R tels que:

1) $\alpha(p) \geqslant \frac{1}{n}$;

2) $H(p)$ n'a de points communs avec aucune des sphères n'appartenant pas à $\sum'$;

3) $H(p)$ a des points communs avec chaque sphère appartenant à $\sum'$.

L'ensemble $F_{n,\Sigma'}$ est un ensemble du type F_σ, parce qu'il est l'intersection de l'ensemble fermé des points de R; où $\alpha(p) \geqslant \frac{1}{n}$, des ensembles fermés, où $H(p)\,S_i \neq 0$, S_i étant une sphère arbitraire de $\sum'$; et des ensembles ouverts des points; où $H(p)\,S_i' = 0$, S_i' étant une sphère arbitraire n'appartenant pas à $\sum'$.

L'ensemble $F_{n,\Sigma'}$ est de première catégorie dans R. En effet; si $p \subset F_{n,\Sigma'}$ et $U(p)$ est un voisinage arbitraire du point p, il existe dans $U(p)$ un point p' tel que

$$\alpha\,(H(p'),\ H(p)) \geqslant \frac{1}{n} \text{ parce que } \alpha(p) \geqslant \frac{1}{n}.$$

Donc, il existe une sphère de $\sum_n$ qui ne contient que les points d'un seul ensemble $H(p')$ ou bien $H(p)$. Cela signifie que p' n'appartient pas à $F_{n,\Sigma'}$, et alors $F_{n,\Sigma'}$ comme un ensemble du type F_σ est de première catégorie dans R.

L'ensemble F de tous les points de discontinuité de R est défini de la manière suivante:

$$F = \sum_{n=1}^{\infty} \sum_{\Sigma' \subset \Sigma_n} F_{n,\Sigma'}$$

où $\sum'$ parcourt tous les sous-ensembles de l'ensemble $\sum_n$ formés des sphères.

Il en résulte que F sera un ensemble du type F_σ et de première catégorie, donc l'ensemble des points de continuité en A sera un ensemble du type G_δ et de seconde catégorie. Ensuite (en vertu du théorème 2) nous voyons que les points de continuité en A, donc aussi les points de continuité locale, existent pour chaque système (1), et d'ailleurs au point de vue descriptif ils présentent la masse fondamentale dans le domaine. Cependant le théorème 1 nous oblige à répondre à la question: peut-être ce ne sont que les points d'unicité en A qui jouissent de cette propriété?

Mais l'exemple connu de M. Lavrentieff [4] qui a construit une équation différentielle $\frac{dy}{dx} = f(x, y)$, telle que tous les points d'un carré sont des points, où l'unicité même locale tombe en défaut, nous apprend que les points, où l'unicité même locale tombe en défaut, peuvent être des points de continuité.

Une étude plus détaillée de cet exemple montre que tous les points de discontinuité sont situés sur une infinité dénombrable de segments. Le problème de la structure de l'ensemble des points de discontinuité dans le cas général est aussi un problème qui reste à resoudre.

Remarquons que tout ce qui à été démontré pour les points de continuité et d'unicité à droite reste valable pour les points de continuité et d'unicité à gauche.

Université d'État
à Sverdlovsk.

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LITTÉRATURE CITÉE

[1] L. E. J. Brouwer, Proc. Konikl. Akad. van Wetensch. te Amsterdam, April 28. 716 (1910); M. Charpantier, C. R., 191, 509 (1930); E. R. v. Kampen, Amer. Journ. of Math., 59 (1937). [2] Hausdorff, Mengenlehre, 2 Aufl., S. 146. [3] Kamke, Acta Math., 58 (1928). [4] M. Lavrentieff, Math. ZS., 23 (1925).

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MATHÉMATIQUES

SUR LA REPRÉSENTATION ANALYTIQUE D'OPÉRATIONS LINÉAIRES MULTIPLICATIVES

Par B. VULICH

(Présenté par A. N. Kolmogoroff, de l'Académie, le 17.VII.1943)

1°. Nous allons considérer les espaces semi-ordonnés concrets suivants:

1) $L^p (p \geqslant 1)$ — l'espace des fonctions réelles à p-ème puissance sommable $x(t)$, définies dans un intervalle fini $[a, b]$.

2) $\tilde{M}$ — l'espace des fonctions réelles mesurables $x(t)$, bornées presque partout dans $[a, b]$.

3) S — l'espace des fonctions réelles $x(t)$, mesurables et finies presque partout dans $[a, b]$ $(a \leqslant t \leqslant b)$.

Dans tous ces espaces la relation $x > 0$ signifie que $x(t) \geqslant 0$ presque partout dans $[a, b]$ et que $x(t)$ n'est pas équivalente à 0. Nous considérons comme l'unité la fonction $x(t) \equiv 1$ (presque partout). Alors le produit des éléments dans ces espaces coïncide avec le produit ordinaire des fonctions. D'ailleurs, le produit des éléments dans L^p existe, si le produit des fonctions correspondantes appartient à L^p.

4) $l^p (p \geqslant 1)$ — l'espace des suites de nombres réels $x = \{\xi^{(k)}\}$ satisfaisant à l'inégalité: $\sum_{k=1}^{\infty} |\xi^{(k)}|^p < +\infty$.

5) m — l'espace des suites bornées de nombres réels $x = \{\xi^{(k)}\}$.

6) s — l'espace de toutes les suites de nombres réels $x = \{\xi^{(k)}\}$.

Dans les derniers trois espaces la relation $x > 0$ signifie que tous les $\xi^{(k)} \geqslant 0$ et, pour un indice k au moins, $\xi^{(k)} > 0$*.

Nous allons indiquer encore un espace:

7) c_0 — l'espace des suites de nombres réels $x = \{\xi^{(k)}\}$ convergeant vers 0 (c.-à-d. $\xi^{(k)} \underset{k \to \infty}{\to} 0$), avec la même définition du signe $>$. La relation $x_n \overset{(o)}{\to} x$ équivaut dans cet espace au fait que $\xi_n^{(k)} \underset{n \to \infty}{\to} \xi^{(k)}$ uniformément par rapport à k ($\xi_n^{(k)}$ sont les termes de x_n).

Dans chacun des espaces de suites énumérés ci-dessus le rôle de l'unité peut jouer tout élément dont tous termes sont > 0. Nous considérons comme l'unité dans m et s l'élément $x = \{\xi^{(k)}\}$, où $\xi^{(k)} = 1$ pour tout k. Mais cette suite n'appartenant pas aux espaces l^p et c_0, nous posons l'unité dans l^p égale à la suite $\{2^{-\frac{k}{p}}\}$, et dans c_0 à $\left\{\frac{1}{k}\right\}$.

Dans les espaces m et s la multiplication des éléments est toujours possible et elle correspond à la multiplication de suites terme à terme, c.-à.-d.

* Tous ces espaces sont étudiés dans un article de Kantorovitch (1). On peut y trouver le sens de la convergence (0) dans chacun de ces espaces.

pour $x=\{\xi^{(k)}\}$ et $y=\{\eta^{(k)}\}$, on a $xy=\{\xi^{(k)}\eta^{(k)}\}$. Dans l^p et c_0 nous obtenons la multiplication «avec un facteur complémentaire», à savoir, dans l^p $xy=\{2^{\frac{k}{p}}\xi^{(k)}\eta^{(k)}\}$ et dans c_0 $xy=\{k\xi^{(k)}\eta^{(k)}\}$. D'ailleurs, pour que le produit existe, il faut et il suffit que la suite de nombres qui représente le résultat de cette multiplication «avec un facteur complémentaire» appartienne à l'espace considéré.

2°. Considérons les opérations linéaires multiplicatives (²) (op. l. m.) définies dans $\tilde{M}$. Nous dirons, comme d'habitude, qu'une fonction réelle $\varphi(s)$ satisfait à la condition (N) (la condition de N. Lusin), si, quel que soit l'ensemble A de mesure égale à 0, l'ensemble des s tels que $\varphi(s)\in A$ a la même mesure. On sait que la fonction $f[\varphi(s)]$, composée de deux fonctions mesurables dont l'intérieure satisfait à la condition (N), est aussi mesurable.

I. La forme générale des op. l. m. $y=U(x)$ transformant $\tilde{M}$ en $\tilde{M}$ est donnée par la formule

$$y(s)=\begin{cases} x[\varphi(s)] \text{ pour } s\in\mathcal{E}, \\ 0 \text{ pour } s\in\overline{\mathcal{E}}, \end{cases} \tag{1}$$

où $\mathcal{E}$ est un ensemble mesurable contenu dans l'intervalle $[a, b]$, et $\varphi(s)$ est une fonction mesurable, définie dans $\mathcal{E}$, dont les valeurs sont aussi de $[a, b]$ et qui satisfait à la condition (N).

II. Les classes des op. l. m. transformant $\tilde{M}$ en $L^p(p\geqslant 1)$ ou en S coïncident avec la classe des op. l. m. qui transforment $\tilde{M}$ en $\tilde{M}$.

Si $y=U(x)$ est une op. l. m. qui transforme $L^p(p\geqslant 1)$ ou S en un des même espaces, alors $y=U(x)$ peut être représentée aussi sous la forme de (1). Mais il est possible, d'ailleurs, que $\varphi(s)$ doit satisfaire à certaines conditions complémentaires, plus fortes que la condition (N). Nous allons indiquer ces conditions qui sont nécessaires et suffisantes. Soit X l'espace où les opérations sont définies, et Y — l'espace où sont situés ses contredomaines.

III. Si $X=Y=L^p(p\geqslant 1)$, alors $\varphi(s)$ doit satisfaire à la condition suivante: quels que soient $\tau_2<\tau_1(-\infty<\tau_2<\tau_1<+\infty)$, on a

$$m\underset{s}{\mathcal{E}}[\tau_2<\varphi(s)\leqslant\tau_1]\leqslant C(\tau_1-\tau_2)(C \text{ est une constante quelconque}).$$

IV. Si $X=L^p(p\geqslant 1)$, $Y=L^q(1\leqslant q<p)$, alors $\varphi(s)$ doit satisfaire à la condition: $\frac{d}{dt}m\underset{s}{\mathcal{E}}[\varphi(s)\leqslant t]\in L^{\frac{p}{p-q}}$.

V. Si $X=L^p(p\geqslant 1)$ ou $X=S$, et $Y=S$, alors il ne faut soumettre $\varphi(s)$ à aucune condition complémentaire.

VI. Il n'existe pas d'op. l. m. qui ne soient pas égales identiquement à zéro dans les cas suivants: a) $X=L^p(p\geqslant 1)$, $Y=L^q(q>p)$ ou $Y=\tilde{M}$, b) $X=S$, $Y=L^q(q\geqslant 1)$ ou $Y=\tilde{M}$.

VII. Le même résultat reste vrai, si X est un des espaces fonctionnels $\tilde{M}$, $L^p(p\geqslant 1)$ ou S, et Y est un des espaces de suites $l^q(q\geqslant 1)$, m, s ou c_0*.

Il est à remarquer que la formule (1) ne représente une opération projective dans $\tilde{M}$ (de même que dans L^p ou S) que quand $\varphi(s)=s$ pour $s\in\mathcal{E}$.

3°. On connaît que chaque opération $y=U(x)$ de la classe H_0^0 transformant L en L peut être représentée sous la forme de

$$U(x)=y(s)=\frac{d}{ds}\int_a^b K(s, t)x(t)\,dt, \tag{2}$$

où le noyau $K(s, t)$ satisfait aux conditions suivantes:

* Cf. (²), le théorème 7.

a) $K(s, t)$ est mesurable superficiellement dans le carré $a \leqslant s,\ t \leqslant b$;
b) pour tout s $K(s, t)$ est mesurable par rapport à t;
c) $K(a, t) \equiv 0$;
d) pour tout t $K(s, t)$ est une fonction à variation bornée par rapport à s et $\sup_t \operatorname{var}_{a \leqslant s \leqslant b} K(s, t) < +\infty$;
e) pour tout $\tau (a \leqslant \tau \leqslant b)$ l'intégrale $\int_0^{\tau} K(s, t)\, dt$ est une fonction absolument continue de s(²).

Posons la question suivante: auxquelles conditions complémentaires doit-on soumettre le noyau $K(s, t)$, pour que la formule (2) donne une op. l. m.?

VIII. Voici ces conditions, nécessaires et suffisantes:

1) quel que soit t, la fonction $L(s, t) = \frac{d}{ds} \int_a^{\tau} K(s, \tau)\, d\tau$ ne prend pour presque tous les s que des valeurs 0 et 1;

2) si $t_1 > t_2$; alors $L(s, t_1) \geqslant L(s, t_2)$ pour presque tous les s.

Nous indiquerons un exemple non-trivial d'une fonction $K(s, t)$ satisfaisant à toutes les conditions mentionnées. Soit $\psi(t)$ $(a \leqslant t \leqslant b)$ une fonction continue, croissante, partout différentiable, dont la dérivée est bornée, et soit d'ailleurs $\psi(a) = a$, et pour tout t $a \leqslant \psi(t) \leqslant b$. Posons

$$K(s, t) = \begin{cases} \psi'(t) & \text{si } s > \psi(t) \\ 0 & \text{si } s \leqslant \psi(t). \end{cases}$$

IX. Pour la représentation d'opérations projectives dans L sous la forme de (2), on obtient, à la place de VIII, la condition suivante: presque partout dans le carré $a \leqslant s,\ t \leqslant b$

$$K(s, t) = \begin{cases} 1 \text{ pour } t \in \mathcal{E},\ s > t \\ 0 \text{ partout ailleurs,} \end{cases}$$

où $\mathcal{E}$ est un ensemble linéaire, mesurable, contenu dans $[a, b]$.

4°. Soient maintenant $X = l^p$, m, s ou c_0, et $Y = L^q$, $\tilde{M}$ ou S. Alors chaque op. l. m. $y = U(x)$ peut être représentée de la manière suivante: il existe des ensembles mesurables disjoints $\mathcal{E}_k \subset [a, b]$ $(k = 1, 2\ldots)$ et des constantes y_k tels que, pour $x = \{\xi^{(k)}\}$

$$U(x) = y(s) = \begin{cases} \xi^{(k)} y_k \text{ si } s \in \mathcal{E}_k (k = 1, 2\ldots) \\ 0 \text{ partout ailleurs.} \end{cases} \tag{3}$$

Prenons, d'abord, $X = l^p (p \geqslant 1)$. Alors $y_k = 2^{\frac{k}{p}}$.

X. Si $Y = S$, on ne doit soumettre les ensembles $\mathcal{E}_k$ à aucune condition complémentaire. La formule (3), où les y_k ont des valeurs indiquées, donne la forme générale des op. l. m.

Dans les autres cas les $\mathcal{E}_k$ doivent satisfaire à conditions complémentaires:

XI. Si $Y = L^q (q \geqslant 1)$, alors, pour $q \geqslant p$, $m\mathcal{E}_k \leqslant C \cdot 2^{-\frac{kq}{p}}$, où C est une constante arbitraire, et, pour $q < p$,

$$\sum_{k=1}^{\infty} 2^{\frac{kq}{p-q}} (m\mathcal{E}_k)^{\frac{p}{p-q}} < +\infty.$$

XII. Si $\tilde{Y} = M$, alors c'est seulement un nombre fini de $\mathcal{E}_k$ qui peut avoir la mesure $m\mathcal{E}_k > 0$.

Prenons maintenant $X = c_0$; on a alors $y_k = k$ dans (3). Si $Y = S$ ou $\tilde{M}$, nous obtenons les mêmes résultats que dans X et XII.

XIII. Si $Y = L^q (q \geqslant 1)$, on a la condition suivante: $\sum_{k=1}^{\infty} k^q m\mathcal{E}_k < +\infty$.

5°. Nous allons considérer maintenant les cas où les deux espaces X et Y sont des espaces de suites.

Soit qu'à certains nombres entiers $n_1 < n_2 < \ldots$ (ils peuvent former un ensemble dénombrable, fini ou nul) correspondent les nombres entiers $k_1, k_2, \ldots$ D'ailleurs, les valeurs de k_i peuvent coïncider pour divers i. Alors la forme générale des op. l. m. $y = U(x)$ peut être donnée pour $x = \{\xi^{(k)}\}$ sous la forme de

$$y = U(x) = \{\eta^{(n)}\}, \quad \text{où } \eta^{(n)} = \begin{cases} \alpha_i \xi^{(k_i)} & \text{pour } n = n_1, n_2, \ldots \\ 0 & \text{pour autres } n \end{cases} \tag{4}$$

(α_i sont quelques constantes dont les valeurs dépendent du choix de X et Y).

Considérons tous les nombres entiers k. A chaque n_i correspond une des valeurs de k, notamment k_i. Mais non pas inversement; chaque k ne trouve pas un correspondant n_i. Nous appellerons «essentielles» les valeurs de k que l'on trouve parmi les k_i. Pour chaque valeur essentielle de k, nous désignerons par N_k le plus petit des n_i correspondant à ce k. Ensuite, si pendant la sommation de quelques expressions par rapport à l'indice k on ne doit se borner qu'aux valeurs essentielles de k, alors nous désignons la somme considérée par le signe *: $\sum^*$ (s'il n'existe aucune valeur essentielle de k, on doit comprendre $\sum^* = 0$).

Nous indiquerons, quelles valeurs doivent avoir les α_i, et à quelles conditions doivent satisfaire les valeurs essentielles de k pour que (4) représente la forme générale des op. l. m. dans divers cas concrets.

XIV. $X = l^p (p \geqslant 1)$, $Y = l^q (q \geqslant 1)$. On a dans ce cas $\alpha_i = 2^{\frac{k_i}{p} - \frac{n_i}{q}}$ et d'ailleurs: a) si $q \geqslant p$; alors on a pour toutes les valeurs essentielles de k: $\frac{kq}{p} \leqslant N_k + C$, où C est une constante; b) si $q < p$, alors on a

$$\sum_{k=1}^{\infty} {}^* 2^{\frac{kq - pN_k}{p-q}} < +\infty.$$

XV. $X = l^p (p \geqslant 1)$, $Y = s$ ou m. On a dans ce cas $\alpha_i = 2^{\frac{k_i}{p}}$. D'ailleurs, si $Y = m$, il ne peut exister qu'un nombre fini de valeurs essentielles de k, et si $Y = s$, on n'a besoin d'aucune condition complémentaire.

XVI. $X = l^p (p \geqslant 1)$, $Y = c_0$. On a dans ce cas $\alpha_i = \frac{1}{n_i} \cdot 2^{\frac{k_i}{p}}$ et, d'ailleurs, $k \leqslant p\left(C + \frac{\log N_k}{\log 2}\right)$ pour toute valeur essentielle de k (C est une constante quelconque).

XVII. $X = Y = c_0$. On a dans ce cas $\alpha_i = \frac{k_i}{n_i}$ et, d'ailleurs, $k \leqslant CN_k$ pour toutes les valeurs essentielles de k.

XVIII. $X = c_0$, $Y = l^q (q \geqslant 1)$. On a dans ce cas $\alpha_i = k_i 2^{-\frac{n_i}{q}}$ et, d'ailleurs,

$$\sum_{k=1}^{\infty} {}^* k^q 2^{-N_k} < +\infty.$$

XIX. $X = c_0$, $Y = s$ ou m. On a dans ce cas $\alpha_i = k_i$, et les conditions complémentaires pour les valeurs essentielles de k sont les mêmes que dans XV.

Il est évident que, si $X = Y$, la formule (4) ne représente une opération projective que dans le cas, où $k_i = n_i$ pour tout i.

La Marine Baltique.

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LITTÉRATURE CITÉE

[1] L. Kantorovitch, Rec. Math., 2 (44), 121—168 (1937). [2] B. Vulich, C. R. Acad. Sci. URSS, XLI, No. 4 (1943). [3] L. Kantorovitch et B. Vulich, Compos. Math., 5, 119—167 (1937).

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MATHÉMATIQUES

SUR LES CONGRUENCES DES COURBES PLANES ALGÉBRIQUES

Par V. RISCHKOV

(Présenté par N. N. Lusin, de l'Académie, le 23. VII. 1943)

1. Nous considérons les congruences (P) des courbes planes algébriques P assujetties à la condition: la courbe P ne passe pas par le point M de la surface (M) qui est l'enveloppe de la famille des plans sur lesquels sont situées les courbes P de la congruence (P). Rapportons la congruence (P) aux paramètres asymptotiques de la surface (M), u et v—au tétraèdre mobile $T(MM_1M_2M_3)$ que nous définissons de la manière suivante: le sommet M décrit la surface (M); M_1 et M_2 sont les points d'intersection de la polaire du point M par rapport à la courbe P avec les tangentes asymptotiques de (M); la droite MM_3 correspond polairement à M_1M_2 par rapport à la surface de Lie du second ordre osculatrice à (M). Alors les équations du mouvement du tétraèdre T peuvent être réduites à la forme(1):

$$\begin{aligned} M_u &= aM + M_1 & M_v &= bM + M_2 \\ M_{1u} &= a_1^0 M - aM_1 + \beta M_3 & M_{1v} &= b_1^0 M + bM_1 + M_3 \\ M_{2u} &= a_2^0 M + aM_2 + M_3 & M_{2v} &= b_2^0 M + \gamma M_1 - bM_2 \\ M_{3u} &= a_3^0 M + a_3^1 M_1 + a_3^2 M_2 - aM_3 & M_{3v} &= b_3^0 M + b_3^1 M_1 + b_3^2 M_2 - bM_3. \end{aligned}$$

En coordonnées locales, définies pour un point N arbitraire au moyen de l'égalité $N = tM + xM_1 + yM_2 + zM_3$, l'équation P peut être écrite dans la forme:

$$\sum_{k=2}^{n} t^{n-k} \sum_{i+j=k} a_{ij} x^i y^j - t^n = 0.$$

Alors les foyers de la courbe P peuvent être déterminés comme ses points d'intersection avec une certaine courbe bien déterminée d'ordre $n+1$ ayant un point double en M. L'équation de cette courbe S est:

$$\sum_{k=2}^{n+1} t^{n-k+1} \sum_{i+j=k} c_{ij} x^i y^j = 0,$$

où

$$c_{ij} = \frac{\partial a_{i-1,j}}{\partial u} - \frac{\partial a_{i,j-1}}{\partial v} - \beta(j+1)\, a_{i-2,\,j+1} + \gamma(i+1)\, a_{i+1,\,j-2} +$$

$$+ 2a(i-1)\, a_{i-1,\,j} - 2b(j-1) a_{i,\,j-1} + (j-i)\, a_{ij} - a_1^0 (n-i-j+2)\, a_{i-2,\,j} +$$

$$+ b_2^0 (n-i-j+2)\, a_{1,\,j-2} + (b_1^0 - a_2^0)(n-i-j+2)\, a_{i-1,\,j-1}.$$

En considérant cette équation nous trouverons que les directions des tangentes à S au point double M sont conjuguées sur la surface (M) (ou bien M est un point multiple d'ordre supérieur à 2 pour S) dans le cas et dans ce cas seulement où la congruence (M_1M_2) des polaires est conjuguée à la surface (M). En particulier, en supposant que la congruence (P) est formée par des courbes situées sur une même surface d'ordre n, nous trouverons que la congruence des polaires des points d'une surface (M) arbitraire par rapport aux sections d'une surface algébrique quelconque par les plans tangents à (M), est toujours conjuguée à (M).

Si la congruence (M_1M_2) est conjuguée à (M), on peut toujours choisir P de telle manière que la multiplicité du point M de S augmente.

On peut poser le problème sur la coïncidence des directions focales correspondant aux foyers différents de P. Si n directions focales coïncident, on peut mener une surface d'ordre n par deux courbes voisines P et P' convenablement choisies. Ce cas est caractérisé par la décomposition de la courbe S; l'une des composantes est une droite passant par M, conjuguée à la direction focale et sur laquelle sont situés les foyers correspondants dont le nombre est égal à n.

Supposons que la courbe S est composée de $n+1$ droites de cette nature, c'est-à-dire les directions focales se décomposent en $n+1$ groupes dont chacun est formé de n directions focales qui coïncident entre elles. Une telle congruence est déterminée d'une manière unique par une congruence des polaires convenablement choisies [et qui est nécessairement conjuguée à (M)] et par le choix des a_{ii} — coefficients de P à indices égaux. Indiquons le type spécial suivant des congruences de cette nature. Soit P_1 la section d'une surface fixe d'ordre $2k$ par un plan tangent à (M). Prenons pour P une courbe quelconque du faisceau ayant des centres multiples d'ordre k aux points d'intersection de P_1 avec les tangentes asymptotiques de (M). Alors la congruence (P) jouira de la propriété indiquée, et d'ailleurs parmi les directions focales multiples d'ordre $2k$ il y en aura $2(k-1)$ qui coïncident avec MM_1 et MM_2, et les trois dernières seront conjuguées aux trois directions hyperosculatrices d'une certaine surface du second ordre osculatrice à (M). Parmi ces congruences P on peut choisir ∞' congruences pour lesquelles ces directions sont les directions de Segré de la surface (M).

Dans le cas général la variation de la courbe P entraîne la variation de la courbe S. Supposons qu'en chaque plan tangent à (M) il existe deux courbes P et P_1 ayant une polaire commune par rapport à M. Considérons une courbe arbitraire du faisceau $P+\lambda P_1$, où λ est constant pour tous les points de la surface (M). Alors, si S et S_1 correspondent à P et P_1, il correspond à cette courbe $S+\lambda S_1$. On peut considérer une courbe P'' appartenant au premier faisceau et ayant en M un point double, et une courbe S'' appartenant au second faisceau et telle que ses tangentes en M sont conjuguées sur la surface (M) [on suppose que la congruence des polaires n'est pas conjuguée à la surface (M)]. Il y a une relation intime entre les courbes P'' et S'' caractérisant les faisceaux P et S. Par exemple, la courbe S'' a en M un point multiple d'ordre supérieur à deux dans le cas et dans ce cas seulement où les tangentes à P'' en M coïncident avec les tangentes asymptotiques à (M).

2. Indiquons maintenant certains problèmes qui se rapportent au cas des courbes du second ordre.

Soient donnés (M), (M_1M_2) non-conjuguée à (M) et deux directions tangentes à S (courbe du troisième ordre) en (M); ces données déterminent une congruence (P) à un choix arbitraire près des courbes P qui sont ses génératrices dans les faisceaux à centres immobiles et situés sur les tangentes asymptotiques à (M). Quel que soit le choix de la courbe P dans un tel faisceau, les trois points d'intersection de S avec la polaire M_1M_2 sont ou bien les trois points d'intersection des tangentes aux directions hyperosculatrices d'une surface quelconque du second ordre osculatrice à (M) ou bien

non — cette propriété ne dépend que des points d'intersection de P avec les tangentes asymptotiques et de la polaire M_1M_2.

Il est facile à montrer que les tangentes à S en M sont dirigées vers les foyers du rayon M_1M_2 de la congruence (M_1M_2) dans le cas et dans ce cas seulement où la courbe P est tangente aux deux tangentes asymptotiques de (M).

La propriété S précédemment indiquée est ici vérifiée.

Introduisons les définitions suivantes: (K_1) est un complexe linéaire contenant les droites MM_1, MM_3 ef ayant un contact du second ordre le long de la direction $v = \text{const}$ avec la congruence (M_1M_2). On définit d'une manière analogue le complex (K_2). Dans les nuls-systèmes de ces complexes aux points de $M_1 + \lambda M_2$ correspondent les plans n_1 et n_2. Ces plans coïncident pour deux points N_1 et N_2. Si ces points sont conjugués respectivement avec les foyers de M_1M_2, nous dirons pour abréger que la congruence (M_1M_2) appartient à la classe (A).

On peut montrer qu'une congruence (P) à quatre foyers situés sur les tangentes asymptotiques à (M) peut être construite dans le cas et dans ce cas seulement où la congruence des polaires appartient à la classe (A). D'ailleurs, pour une congruence donnée de ce type M_1M_2 la congruence (P) jouissant de la propriété indiquée est définie d'une manière unique.

La congruence des directrices de Wilczinski est toujours une congruence (A). La congruence (P) liée à (A) est la congruence des courbes considérée par Buchin Su [2].

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LITTÉRATURE CITÉE

[1] С. П. Фиников, Проэктивно-дифференциальная геометрия, М., 1937. [2] Buchin Su, Tohoku Math. Journ., 40 (1935).

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MATHÉMATIQUES

SUR LE PROBLÈME DE LA DÉTERMINATION DE LA BORNE D'UN POLYNÔME INTERPOLATOIRE

Par A. TOURETZKY

(Présenté par S. L. Soboleff, de l'Académie, le 22. VII. 1943)

K. Grandjot [1] a étudié le problème de la détermination de la borne supérieure du module d'un polynôme quand ce module est au plus égal à 1 aux points dont les abscisses sont celles de Tchebycheff. S. Bernstein a étudié le même problème pour les polynômes dont les modules ne surpassent pas 1 aux points dont les abscisses sont proches à celles de Tchebycheff. L'auteur a déterminé la borne supérieure du module d'un polynôme quand ce module ne surpasse pas 1 en des points équidistants.

Dans la présente Note on résout le problème suivant proposé par S. Bernstein: déterminer la borne supérieure sur $[-1, 1]$ du module d'un polynôme quand ce module ne surpasse pas 1 en des points équidistants des intervalles $(-1, -\alpha)$ et $(\alpha, 1)$ pour $0 < \alpha < 1$, et trouver la valeur de α pour laquelle cette borne supérieure est minimale.

En s'appuyant sur la formule interpolatoire, on peut présenter le polynôme dans la forme:

$$P_{2n+1}(x) = \sum_{k=0}^{n} (-1)^{k-1} \frac{A(x) P_{2n+1}(x_k)}{\beta^n k!\,(n-k)!\,(x-x_k)(2-k\beta)\ldots(2-k+n\beta)} + \\ + \sum_{k=0}^{n} (-1)^k \frac{A(x) P_{2n+1}(-x_k)}{\beta^n k!\,(n-k)!\,(x+x_k)(2-k\beta)\ldots(2-k+n\beta)},$$

où $A(x) = (x^2 - x_0^2)\ldots(x^2 - x_n^2)$, $\quad x_k = -1 + k\beta \quad (k = 0, 1, \ldots, n), \quad \beta = \frac{1-\alpha}{n}$.

Comme $|P_{2n+1}(\pm x_k)| \leqslant 1$, on a

$$|P_{2n+1}(x)| \leqslant \sum_{k=0}^{n} \frac{|A(x)|}{|x-x_k|\,\beta^n k!\,(n-k)!\,(2-k\beta)\ldots(2-k+n\beta)} + \\ + \sum_{k=0}^{n} \frac{|A(x)|}{|x+x_k|\,\beta^n k!\,(n-k)!\,(2-k\beta)\ldots(2-k+n\beta)} = Q_{2n+1}(x).$$

Il suffit d'étudier l'allure de $Q_{2n+1}(x)$ sur $(-1, 0)$, parce que

$$Q_{2n+1}(-x) = Q_{2n+1}(x).$$

Supposons d'abord que $x_l < x < x_{l+1}$, où l est égal à l'un des nombres

$0, 1, \ldots, n-1$. Alors en posant $x = x_l + \Theta\beta$, où $0 < \Theta < 1$, nous aurons:

$$Q_{2n+1}(x) = \frac{\Gamma(\Theta+l+1)\,\Gamma(n+1-l-\Theta)\,\Gamma\left(\frac{2n}{\gamma}-\Theta-l+1\right)}{\Gamma(\Theta)\,\Gamma(1-\Theta)\,\Gamma\left(\frac{2n}{\gamma}-\Theta-n-l\right)} \times$$

$$\times\left[\sum_{k=0}^{l}\frac{1}{k!\,(n-k)!}\,\frac{1}{\Theta-k+l}\,\frac{\Gamma\left(\frac{2n}{\gamma}-k-n\right)}{\Gamma\left(\frac{2n}{\gamma}-k+1\right)}+\right.$$

$$+\sum_{k=l+1}^{n}\frac{1}{k!\,(n+k)!}\,\frac{1}{k-\Theta-l}\,\frac{\Gamma\left(\frac{2n}{\gamma}-k-n\right)}{\Gamma\left(\frac{2n}{\gamma}-k+1\right)}+$$

$$\left.+\sum_{k=0}^{n}\frac{1}{k!\,(n-k)!}\,\frac{1}{\frac{2n}{\gamma}-k-\Theta-l}\,\frac{\Gamma\left(\frac{2n}{\gamma}-k-n\right)}{\Gamma\left(\frac{2n}{\gamma}-k+1\right)}\right]=U_l(\Theta),$$

où $\gamma = 1-\alpha$.

En considérant la différence

$$U_{l+1}(\Theta)-U_l(\Theta)=\frac{(n+1)\,\Gamma(\Theta+l+1)\,\Gamma(n-\Theta-l)\,\Gamma\left(\frac{2n}{\gamma}-\Theta-l\right)}{\Gamma(\Theta)\,\Gamma(1-\Theta)\,\Gamma\left(\frac{2n}{\gamma}-\Theta-n-l\right)}\times$$

$$\times\left[\left(\frac{2n}{\gamma}+1\right)\left(\frac{2n}{\gamma}-2n-1\right)\sum_{k=0}^{l}\frac{1}{k!\,(n-k)!}\,\frac{\Gamma\left(\frac{2n}{\gamma}-k-n\right)}{\Gamma\left(\frac{2n}{\gamma}-k+1\right)}-\right.$$

$$-\left(\frac{2n}{\gamma}-2\Theta-2l-1\right)\sum_{k=0}^{n-l-1}\left(\frac{2n}{\gamma}-2n+2k-1\right)\times$$

$$\left.\times\frac{1}{k!\,(n-k+1)!}\,\frac{\Gamma\left(\frac{2n}{\gamma}-2n+k-1\right)}{\Gamma\left(\frac{2n}{\gamma}-n+k+1\right)}\right],$$

il est facile à voir que si l'on fixe Θ sur $(0, 1)$ et si l'on fait croître l, la fonction $U_l(\Theta)$ décroît d'abord à partir de $U_0(\Theta)$, jusqu'à ce qu'elle atteint son minimum pour une ou deux valeurs de l $(0 < l < n-1)$, et ensuite elle croît jusqu'à $U_{n-1}(\Theta)$. Ainsi le plus grand des maxima de $U_l(\Theta)$ sera ou bien le maximum de $U_0(\Theta)$ ou bien le maximum de $U_{n-1}(\Theta)$.

En calculant ces maxima on trouve

$$\max U_0(\Theta) \sim \frac{\sqrt{2\pi}}{4e}\,\frac{\gamma}{(1-\gamma)^{3/4}}\,\frac{1}{\sqrt{n\log n}}\left[\frac{2^{\frac{2}{\gamma}}(1-\gamma)^{\frac{1}{\gamma}-1}}{(2-\gamma)^{\frac{2}{\gamma}-1}}\right]^n,$$

$$\max U_{n-1}(\Theta) \sim \frac{\sqrt{\pi}}{2e}\,\frac{\sqrt{\gamma}\sqrt{1+\sqrt{1-\gamma}}}{(1-\gamma)^{1/4}}\,\frac{1}{\sqrt{n\log n}}\left[\frac{(2-\gamma)^{\frac{2}{\gamma}-1}}{2^{\frac{2}{\gamma}-2}(1-\gamma)^{\frac{1}{\gamma}-1}}\right]^n.$$

Il est facile à remarquer que pour chaque γ fixe sur $(0, 1)$ la valeur asymptotique du maximum de $U_0(\Theta)$ surpasse la valeur asymptotique du maximum de $U_{n-1}(\Theta)$. Donc, la borne supérieure de $|P_{2n+1}(x)|$ sur les segments $[-1, -\alpha]$ et $[\alpha, 1]$ est asymptotiquement égale au maximum de $U_0(\Theta)$. Cette borne supérieure est réellement atteinte pour le polynôme qui prend alternativement les valeurs -1 et $+1$ aux points interpolatoires, au point

dont l'abscisse est asymptotiquement égale à $-1+\frac{\gamma}{n\log n}$ et au point symétrique par rapport à l'origine.

Déterminons maintenant la borne supérieure de $|P_{2n+1}(x)|$ sur $[-\alpha, \alpha]$. Il est facile à voir que sur ce segment on a

$$|P_{2n+1}(x)| \leqslant 2\sum_{k=0}^{n} \frac{(1-k\beta)[1-x^2]\ldots[(1-\overline{k-1}\beta)^2-x^2]\,[(1-\overline{k+1}\beta)^2-x^2]\ldots[(1-n\beta)^2-x^2]}{\beta^n\, k!\,(n-k)!\,(2-k\beta)\ldots(2-\overline{k+n}\beta)} = V(x)$$

Quand x^2 croît de 0 à α^2, la fonction $V(x)$ décroît, donc le maximum de $V(x)$ sur $-\alpha \leqslant x \leqslant \alpha$ est égal à $V(0)$.

Le calcul donne

$$V(0) \sim \frac{(1-\gamma)^{1/4}\sqrt{1+\sqrt{1-\gamma}}}{2\sqrt{\gamma}} \left[\frac{1}{\gamma(1-\gamma)^{\frac{1}{\gamma}-1}}\right]^n.$$

Re arquons que la borne supérieure de $|P_{2n+1}(x)|$ sur le segment $[-\alpha, \alpha]$ est aussi réellement atteinte pour le polynôme qui prend les valeurs -1 et $+1$ alternativement aux points interpolatoires.

Pour déterminer la borne supérieure de $|P_{2n+1}(x)|$ sur tout le segment $[-1, 1]$ il nous reste à comparer les valeurs asymptotiques des quantités $V(0)$ et $\max U_0(\Theta)$. A cet effet considérons leur rapport

$$\frac{V(0)}{\max U_0(\Theta)} \sim e\sqrt{\frac{2}{\pi}}\,\frac{(1-\gamma)\sqrt{1+\sqrt{1-\gamma}}}{\gamma^{3/2}}\sqrt{n\log n}\left[\frac{(2-\gamma)^{\frac{2}{\gamma}-1}}{\gamma 2^{\frac{2}{\gamma}}(1-\gamma)^{\frac{2}{\gamma}-2}}\right]^n.$$

Quand γ croît de 0 à 1 l'expression

$$\frac{(2-\gamma)^{\frac{2}{\gamma}-1}}{\gamma 2^{\frac{2}{\gamma}}(1-\gamma)^{\frac{2}{\gamma}-2}}$$

décroît de ∞ à $\frac{1}{4}$ en atteignant la valeur 1 pour $\gamma=\frac{2}{3}\left(\alpha=\frac{1}{3}\right)$. Donc, pour $0<\gamma\leqslant\frac{2}{3}\left(\frac{1}{3}\leqslant\alpha<1\right)$ la valeur asymptotique de $V(0)$ surpasse la valeur asymptotique du maximum de $U_0(\Theta)$. Donc, dans ce cas la borne supérieure de $|P_{2n+1}(x)|$ sur le segment $[-1, 1]$ coïncide avec $V(0)$. Et pour $\frac{2}{3}\leqslant\gamma<1\left(0<\alpha\leqslant\frac{1}{3}\right)$ la valeur asymptotique de $V(0)$ est inférieure à la valeur asymptotique du maximum de $U_0(\Theta)$; donc, dans ce cas la borne supérieure de $|P_{2n+1}(x)|$ sur le segment $[-1, 1]$ coïncide avec le maximum de $U_0(\Theta)$.

La borne supérieure de $|P_{2n+1}(x)|$ *sur le segment* $[-1, 1]$ *sera minimale pour* $\gamma=\frac{2}{3}\left(\alpha=\frac{1}{3}\right)$ *et elle est asymptotiquement égale à*

$$\frac{\sqrt{1+\sqrt{3}}}{2\sqrt{2}}\left(\frac{3\sqrt{3}}{2}\right)^n.$$

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LITTÉRATURE CITÉE

[1] K. Grandjot. Jahresber. der D. Math. Ver., 34, 80 (1925). [2] Serge Bernstein, Bull. Acad. Sci. URSS, série mathem., 1025 (1931). [3] А. Х. Турецкий, Труды Витебск. пединститута, III, 117 (1940).

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PHYSICS

ON THE INVESTIGATION OF ELECTRONIC-VIBRATIONAL LEVELS IN COMPLICATED MOLECULES

By V. HERMAN

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The states of complicated molecules of a very wide class, especially of those of hydrocarbons; are determined by a set of «canonical structures», i. e. of different independent spin schemes which are possible for a given total spin. Their number for a molecule having N electrons, responsible for binding, and total spin S equals (1)

$$\frac{N!\,(2S+1)}{\left(\frac{N}{2}-S\right)!\left(\frac{N}{2}-S+1\right)!}.$$

In the case of the benzene molecule ($N=6$, $S=0$) which has been studied theoretically in detail by Sklar and others (2, 3) within recent years there are five canonical structures.

This enabled the authors mentioned above, by means of a straightforward analysis of symmetry properties of these structures, to determine possible kinds of benzene states and of transitions between them, thus giving a theoretical interpretation of numerous (over 500) experimental investigations of this molecule.

However, as the above formula clearly shows, the number of canonical structures increases rapidly with N. In some cases of complicated molecules, which nevertheless may yet be relatively very simple with respect to their chemical properties, this number of structures becomes immense [for instance, for a molecule of pyrene $C_{16}H_{10}$ ($N=16$, $S=0$) · 4 benzene rings, symmetry D_{2h}, their number amounts to 1430]. The straightforward analysis of such a huge number of canonical structures may be considered as absolutely impossible. Therefore, it becomes necessary to develop some method enabling to determine possible types of levels and transitions for a molecule without turning to straightforward analysis of the entire huge set of its canonical structures.

While endeavouring to solve this problem, one may first of all pay attention to the fact that the determination of possible types of levels of a molecule, the state of which is determined by a given set of canonical structures compatible with a given total spin, means, in other words, the selection from all the irreducible representations of substitution group of Schrödinger equation (this group coinciding with symmetry group of the molecule) of such representations which for a given total spin are allowed by the Pauli principle. This selection can be performed in a manner similar to that used for atoms, with the only difference that the part of characters of the irreducible representations of rotation group will now be played by irreducible representation of a symmetry group of the molecule Γ_i.

For a given S only those are permitted, for which their direct product $\Gamma_i \times \Delta_s$ (Δ_s is the permutation group of spin functions of those electrons which are responsible for molecular binding in the state with total spin S) involves an antisymmetrical representation U, or, and this amounts to the same, which are contained in $\Delta_s \times U$.

The character $\chi(P_s)$ of permutation $P_s(c)$ which corresponds to a given element of the symmetry group of the molecule in the representation of $\Delta_s \times U$, is determined by the equation of Frobenius [4] [which is used in the theory of the atom [5,6] as well] as a coefficient before the $k = \frac{1}{2}N - S$ power of x with the aid of the following generating function:

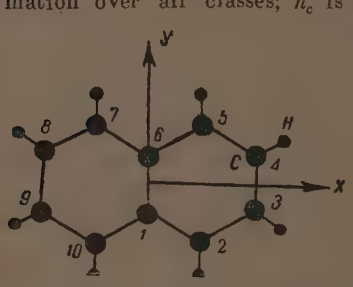
$$(-1)^{N-\rho}(1-x)\prod_{i=1}^{\rho}(1+x^{a_i}) = \sum_{k}\chi^{(k)}(P_s(c))\,x^k.$$

Here a_i is the number of terms in i-th cycle ($i = 1, 2 \ldots \rho$); ρ, the number of cycles; $\sum_k$ designates summation over $k = \frac{1}{2}N - S$.

Now, if this equation is made use of for the determination of character $\chi(P_s(c))$ corresponding to all elements of symmetry group of the molecule, and development over the irreducible representations Γ_i is performed:

$$\Gamma_s = \sum_i a_i \Gamma_i,$$

where $a_i = \frac{1}{h}\sum_c h_c \chi(P_s(c))\,\chi_i(c)$; h is the order of the group; $\sum_c$ means summation over all classes; h_c is the number of elements in the class; $\chi(P_s(c))$ is the character corresponding to $P_s(c)$ for the element of symmetry group of class c (correspondingly, $\chi_i(c)$, the character of Γ_i), as determined by the above formula, then, in accordance with the above reasoning, we come to the permitted types of electron levels. Namely, only those levels are allowed, for which $a_i \neq 0$.



As an example we consider the molecule of naphtalene $C_{10}H_8$ (two benzene rings) which comes after benzene in the order of complexity. Spectra of this molecule were thoroughly studied. Symmetry of $C_{10}H_8$, $D_{2h} = D_2 \times I$. The number of irreducible representations Γ_i of this group, which in this case coincides with the number of elements, equals eight ($i = 1, 2, \ldots, 8$). The corresponding table of characters has been computed by Wigner [7,8].

In order to determine the irreducible representations (i. e. the types of states) compatible with Pauli principle we shall find $\chi(P_s(c))$. For $C_{10}H_8$, $N = 10$, the total spin in the ground state is $S = 0$ (this follows immediately from the diamagnetical properties of naphtalene as well). Let us determine, for instance, $\chi(P_{s=0}(c_2^y))$ corresponding to the element c_2^y (oy is the axis of second order) of the group D_{2h}. It corresponds to the substitution $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 1 & 10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 \end{pmatrix} \rightarrow (1)\ (6)\ (5, 2)\ (3, 4)\ (10, 7)\ (8, 9)$, i. e. it splits up into two single-term and four two-term cycles: $\alpha_1 = \alpha_2 = 1$, $\alpha_3 = \alpha_4 = \alpha_5 = \alpha_6$, $\rho = 6$. The character $\chi(P_{s=0}(c_2^y))$ we are determining will be found as a coefficient before the $k = \frac{1}{2}N - S = 5$ power of x in the expression: $(-1)^{10-6}(1-x)(1+x)^2(1+x^2)^4$, and turns out to equal 2. Similar calcula-

tions for all the elements of D_{2h} lead to the following table for $\chi(P_{s=0}(c))$:

$$E \quad c_2^z \quad c_2^y \quad c_2^x \quad I \quad \sigma^x \quad \sigma^y \quad \sigma^z$$

$$\chi(P_{s=0}(c)) \quad 42 \quad 10 \quad 2 \quad 10 \quad 10 \quad 42 \quad 10 \quad 2$$

Now, having found the values of $\chi(P_{s=0}(c))$ and χ_i, we determine $\Gamma_{s=0} = 16\Gamma_1 + 10\Gamma_3 + 10\Gamma_6 + 6\Gamma_8$ (thus, the levels Γ_2, Γ_4, Γ_5 and Γ_7 are forbidden as being incompatible with the Pauli principle). The components of dipole moment $P = \sum er$ of the molecule $C_{10}H_8$ transform respectively as follows: $P_x \to \Gamma_6$, $P_y \to \Gamma_8$, $P_z \to \Gamma_7$. Herefrom we find that for the purely electronic transition, with the positions of nuclei fixed, only transitions to the levels of the types of Γ_6 (here $\{P_x\}_{\Gamma_1 \to \Gamma_6} \neq 0$, $\{P_y\}_{\Gamma_1 \to \Gamma_6} = \{P_z\}_{\Gamma_1 \to \Gamma_6} = 0$) and Γ_8 ($\{P_y\}_{\Gamma_1 \to \Gamma_6} \neq 0$, $\{P_x\}_{\Gamma_1 \to \Gamma_8} = \{P_z\}_{\Gamma_1 \to \Gamma_8} = 0$) are permitted.

Let now nuclei vibrations be considered first, and subsequently, the possible electronic-vibrational levels arising herefrom. For purely vibrational transitions, the state of electrons being fixed, the number of different possible vibrations b_i of given type $\gamma_1\gamma_2 \ldots \gamma_8$ (characters γ_i are identical with Γ_i) equals, as shown by Wigner [6-8],

$$b_i = \frac{1}{h} \sum_c h_c \chi_{\text{vibr}}(c)\, \chi_i(c),$$

where

$$\chi_{\text{vibr}}(c) = \begin{cases} (U_{c_\varphi} - 2)(1 + 2\cos\varphi_{c_\varphi}), \\ -U_{s_\varphi}(1 + 2\cos\varphi_{s_\varphi}) \end{cases}$$

while U_{c_φ} and U_{s_φ} are the number of nuclei remaining in their places when the operations c_φ (rotation through the angle φ) and s_φ (inversion axis) are performed, respectively. For a molecule $C_{10}H_8$ the total number b_i of completely symmetrical vibrations of the type γ_1 is $b_1 = 5\,(9)$, $b_2 = 4\,(2)$, $b_3 = 4\,(8)$, $b_4 = 1\,(3)$, $b_5 = 2\,(4)$, $b_6 = 4\,(8)$, $b_7 = 2\,(4)$, $b_8 = 4\,(8)$ (the figures in parentheses mean the number of vibrations of the whole molecule, including the protons H as well as the C nuclei). The full number of vibrations is 24 (48). Not a single of these vibrations is degenerated. For a purely vibrational transition (the state of electrons being fixed) only the following vibrations will be active in infra-red: $b_6 = 4\,(8)$ vibrations of the type γ_6 ($\{P_x\}_{\Gamma_1 \times \gamma \to \Gamma_1 \times \gamma_6} \neq 0$ or, briefly, $P_x \neq 0$, $P_y = P_z = 0$), $b_8 = 4\,(8)$ vibrations γ_8 ($P_y \neq 0$, $P_x = P_z = 0$), $b_7 = 2\,(4)$ vibrations γ_7 ($P_z \neq 0$, $P_x = P_y = 0$).

In a purely vibrational Raman spectrum [components of the polarizability tensor c_{ik}, $(i \neq k)$, transform respectively as follows: c_{11}, c_{22}, $c_{33} \to \gamma_1$, $c_{12} \to \gamma_3$, $c_{13} \to \gamma_4$, $c_{23} \to \gamma_2$] activity will be shown by $b_1 = 5\,(9)$ completely symmetrical vibrations γ_1 [in which case $\{c_{11}\}_{\Gamma_1 \times \gamma_1 \to \Gamma_1 \times \gamma_1} \neq 0$ or, briefly, $c_{11} \neq 0$, and correspondingly $c_{22} \neq 0$, $c_{33} \neq 0$, while all $c_{ik} = 0$, $(i \neq k)$]. For these lines the degree of depolarization ρ when the incident light is polarized, and $\rho_n = \frac{2\rho}{1+\rho}$ when the light is not polarized, and likewise the reversion coefficient P vanish; $b_2 = 4\,(2)$ ($c_{23} \neq 0$, all other $c_{ik} = 0$), $b_3 = 4\,(8)$ ($c_{12} \neq 0$), $b_4 = 1\,(3)$ ($c_{13} \neq 0$) not completely symmetrical vibrations of the types γ_2, γ_3, γ_4. For all of them $\rho = \frac{3}{4}$, $\rho_n = \frac{6}{7}$, $P = 6$. The vibrations of the type γ_5 are active neither in infra-red, nor in purely vibrational Raman spectra. We are immediately interested in electronic-vibrational transitions, when the state of electrons and position of nuclei change simultaneously. They are obtained from the integral $\int \psi_{\Gamma_1}(Pl)\, \psi_{\Gamma_i \times \gamma_k}(\nu_k)\, d\tau$, where $\psi_{\Gamma_i \times \gamma_k}(\nu_k)$ is the wave function corresponding to electronic state Γ_i and vibrational state $\gamma_k^{(\nu_k)}$ ($k = 1, 2, \ldots, 8$ is the number of irreducible representations; ν_k, the vibrational quantum number).

Taking into account that for non-degenerated states $\chi_k^{(\nu_k)} = [\chi_k]^{\nu_k}$ (similar

formulae are known for degenerated states as well), we get for $C_{10}H_8$ molecule the following result. The electronic-vibrational lines around the purely electronic transition $\Gamma_1 \to \Gamma_6$ will consist of three groups of lines, differing from each other with respect to polarization. For the first of this group of lines the frequencies may be symbolically written down as follows:

$$\omega\,(\Gamma_1 \to \Gamma_6 \times \gamma_i^{v_i} \times \gamma_i^{v_i}),\ (P_x \neq 0,\ P_y = P_z = 0),\ v_1 = 0, 1, 2 \ldots,\ v_i = 0, 2, 4, 6, \ldots$$
$$(i = 2, 3, \ldots 8),$$

i. e. the purely electronic transition $\Gamma_1 \to \Gamma_6$ ($P_x \neq 0$, $P_y = P_z = 0$) may combine with any of $b_1 = 5\,(9)$ completely symmetrical vibrations, while the states with arbitrary values of vibrational quantum number may be excited. Moreover, the combination with not completely symmetrical vibrations γ_i ($i = 2, 3; \ldots, 8$) is possible, but the vibrational quantum numbers v_i ($i = 2, \ldots, 8$), are necessarily even. For the second group of lines ($P_y \neq 0$, $P_x = P_z = 0$)

$$\omega\,(\Gamma_1 \to \Gamma_6 \times \gamma_3^{v_3} \times \gamma_1^{v_1} \times \gamma_i^{v_i}),\ v_3 = 1, 3, 5, \ldots \text{ (it is necessary that } v_3 \neq 0),$$
$$v_1 = 0, 1, 2, \ldots;\quad v_i = 0, 2, 4, 6 \ldots \quad (i = 2, 4, 5, \ldots, 8);$$

for the third group ($P_x = P_y = 0$, $P_z \neq 0$)

$$\omega\,(\Gamma_1 \to \Gamma_8 \times \gamma_4^{v_4} \times \gamma_1^{v_1} \times \gamma_i^{v_i}),\ v_4 = 1, 3, 5, \ldots\ (v_4 \neq 0),$$
$$v_1 = 0, 1, 2, \ldots,\quad v_i = 0, 2, 4, \ldots \quad (i = 2, 3, 5, \ldots, 8).$$

For this group of lines the intensity may be expected to be negligibly small, because the z axis is perpendicular to the plane of the molecule. Similarly, for the electronic-vibrational transitions grouping around the purely electronic transition $\Gamma_1 \to \Gamma_8$ the following three groups of lines are possible:

$$\omega\,(\Gamma_1 \to \Gamma_8 \times \gamma_3^{v_3} \times \gamma_1^{v_1} \times \gamma_i^{v_i}),\quad (P_x \neq 0,\quad P_y = P_z = 0),\quad v_3 = 1, 3, 5, \ldots\quad (v_3 \neq 0),$$
$$v_1 = 0, 1, 2, \ldots;\quad v_i = 0, 2, 4, \ldots\ (i = 2, 4, 5, \ldots, 8);$$
$$\omega\,(\Gamma_1 \to \Gamma_8 \times \gamma_1^{v_1} \times \gamma_i^{v_i}),\quad (P_x = P_z = 0,\ P_y \neq 0),$$
$$v_1 = 0, 1, 2, \ldots;\quad v_i = 0, 2, 4 \ldots\quad (i = 2, 3, \ldots, 8);$$
$$\omega\,(\Gamma_1 \to \Gamma_8 \times \gamma_2^{v_2} \times \gamma_1^{v_1} \times \gamma_i^{v_i}),\ v_2 = 1, 3, 5, \ldots\ (v_2 \neq 0),$$
$$v_1 = 0, 1, 2, \ldots,\quad v_i = 0, 2, 4, \ldots\quad (i = 3, 4. \ldots, 8)\ (P_z \neq 0,\ P_x = P_y = 0).$$

We wish to point out that since for $C_{10}H_8$ molecule the triplet levels transform precisely according to the same irreducible representations, as the single levels, then even the supposition on the removal of the intercombinational forbiddance [as in the case of benzene (3)] cannot lead to appearance of new types of transitions, differing from those considered above.

Sequence of levels of the types Γ_1, Γ_6, Γ_8, Γ_3 nearest to the ground level may be determined by means of the approximative method of Bloch, which was applied to molecules by Hückel, Lund, Mulliken and others (8-11).

The energy levels corresponding to every single electron are determined from the solution of the following secular equation (10)

$$|\eta_{kl} - x\delta_{kl}| = 0,\ \text{where } k, l \text{ are number of atoms,}$$

$$x = \frac{\varepsilon - \varepsilon^{(0)} - Q}{\alpha},\quad \delta_{kl} = \begin{cases} 1 \text{ for } k = l \\ 0 \text{ for } k \neq l, \end{cases} \qquad \eta_{kl} = \begin{cases} 1 \text{ for neighbouring } k, l, \\ 0 \text{ otherwise.} \end{cases}$$

Here Q is the Coulomb integral; α, the exchange integral; $\varepsilon^{(0)}$, ε, the energies of a single electron in isolated atom and in molecule, respectively.

Solution of the equation shows that the energy level nearest to the fundamental one will be of the type Γ_8, while the two subsequent levels of the type Γ_6. Using the Wigner-Neumann theorem (6,7), according to which the terms corresponding to the single irreducible representation of the same race

do not intersect with each other, we conclude that the levels given by the strict solution will be distributed according to the same sequence.

The above refers to a single molecule fixed in the space. In the case of a molecular crystal, keeping in mind that for the frequency of intramolecular vibrations of nuclei ω_N the equality $\omega_N \gg \omega_L$ holds (ω_L is the frequency of vibrations of simple molecular lattices with respect to each other), we may in the first approximation represent the line frequencies of the crystal spectrum as $\omega_e + \omega_N + \omega_L$ (here ω_e is the frequency of a purely electronic transition within the molecule, $\omega_e \gg \omega_N \gg \omega_L$).

Thus, on the distances $\omega_L \ll \omega_N$ the «crystallic satellites» will become noticeable for the most intensive lines, while the essential features of the distribution of lines of the spectrum remain the same as for the single molecule (neglecting, of course; the rotational structure which will be absent, and omitting the consideration of absolute intensity of the lines; which in the case of a crystal may differ essentially from the case of a single molecule, especially if the light is polarized).

Let us point out that the influence of the crystal may appear also in the possible splitting of levels of a single molecule in the crystal due to the field of all other molecules of the lattice. In the $C_{10}H_8$ crystal this possibility is excluded, since the levels are not degenerated. With respect to any other molecular crystal the possibility of splitting of the levels due to the crystal field may be considered and cleared up by the method similar to that used by Bethe for the atom ([12]). The only difference between Bethe's treatment of splitting of the atomic levels and our case is that in the case of the atom we must reduce the rotation group to the group of symmetry of the crystal field, while for the determination of splitting of the molecular levels it is necessary to reduce the representation of the group of symmetry of molecule to the irreducible representations of the group of symmetry of the crystal. We hope to return to this problem in detail in another work.

The above results are in accordance with the experiment, in particular with a thorough investigation of the spectra of $C_{10}H_8$ crystals at hydrogen temperatures performed by A. F. Prikhotjko. The detailed comparison of the theory with the experiment will be given in the paper mentioned above.

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REFERENCES

[1] W. Heisenberg, ZS. f. Phys., **69**, 287 (1931). [2] A. L. Sklar, Journ. of Chem. Phys., **5**, 669 (1937). [3] H. Sponer, G. Nordheim, A. L. Sklar, E. Teller, ibid., **7**, 207 (1939). [4] Frobenius, Sitzungsber. der Berl. Akad., 516 (1900); 303 (1901). [5] Van der Waerden, Method von Gruppentheorie und Quantenmechanik. 1936. [6] E. Wigner, Gruppentheorie und ihre Anwendung auf die Quantenmechanik der Atomspektren, Braunschweig, 1931. [7] E. Wigner, Nachr. Ges. Wiss. Göttingen, 133 (1930). [8] Rosenthal and G. M. Murphy, Rev. Mod. Phys., **8**, 317 (1936). [9] E. Hückel, ZS. f. Phys., **70**, 204 (1931); **72**, 310 (1931); **76**, 628 (1932). [10] E. London, Journ. de Phys. et Rad., **8**, 397 (1937). [11] R. S. Mulliken, Journ. Chem. Phys., **7**, 356 (1939). [12] H. Bethe, Ann. d. Phys., **3**, 133 (1929).

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TECHNICAL PHYSICS

SCREENS WITH UNEVEN SURFACES

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The brightness of an illuminated or a self-luminescent uneven surface does not obey Lambert's law, even in case it holds true for each of its elementary areas. On the average, depressions look brighter than protuberances. The total brightness of such a surface is in the perpendicular direction greater than at small angles. This serves, for example, to explain the complicated relation between the surface brightness of the Moon and the angle of observation.

The solution of this problem in its general aspect is extremely complicated; but the main peculiar features of the various cases of luminescence permit of a better insight if the case of a surface covered with cone-like depressions be taken as an example.

Let us consider three cases of luminescence of such a surface: that of illumination with visible light, self-luminescence and illumination of a luminescent surface with «red» light (phenomenon of flash).

Fig. 1

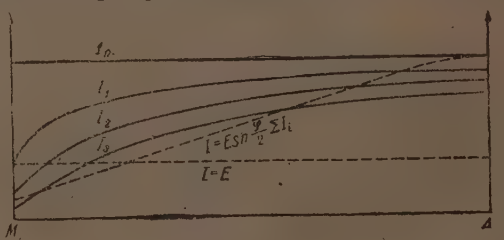

Fig. 2

1. Illumination with Visible Light. Let a light possessing an intensity E fall on a cone in a direction parallel to its axis (Fig. 1). The brightness of the luminescence of the cone I_0 is proportional to the coefficient of scattering p, $\sin\frac{\varphi}{2}$; where φ is the linear angle of the cone and E. I_0 will then be equal to $pE\sin\frac{\varphi}{2}$*.

The ray of light striking point b_0 of the cone is partly being absorbed and partly scattered (let it be according to Lambert's law). Part of the scattered light falls on b_1 and so on. The portion of scattered light δ which

* The coefficient expressing the undifuseness of an incident light beam is neglected.

does not escape through the base of the cone M, is determined by the relation between the solid angle at which the lateral surface of the cone is viewed from point b_1, and 2π. If b_1 is near to M, then $\delta = \frac{\pi - \varphi}{2\pi} \approx \frac{1}{2}$ ($\varphi \ll \pi$). If b_1 is near to A, $\delta = \cos \frac{\varphi}{2}$.

In contrast to I_0, brightness I_1, as due to primarily scattered light, will be irregular, since $I_1 = p\delta I_0$, where δ is a variable quantity. In accordance with what has been said above, I_1 will be at minimum $\left(I_1 = \frac{1}{2} pI_0\right)$ near the base of the cone M, and about twice as great $\left(I_1 = pI_0 \cos \frac{\varphi}{2} \approx pI_0 \text{ for } \varphi \ll \pi\right)$ near the vertex of A (Fig. 2).

Brightness I_2 at any point b_2 due to secondary light scattering can be represented in the form of the product $p\delta\bar{I}_1$, where $\bar{I}_1$ is the mean brightness of the cone efficient for b_2 and due to primarily scattered light. If $\bar{I}_1$ be put equal to the value of I_1 ($I_1 = p\delta I_0$) for point b_2; then $I_2 \approx (p\delta)^2 I_0$.

By reasoning in the same way we obtain that, roughly, the brigthness due to triply scattered light $I_3 \approx (p\delta)^3 I_0$ etc.*

The full brightness may thus be evaluated with the aid of a series

$$I \approx I_0 + I_1 + I_2 + \ldots = I_0 \frac{1}{1 - p\delta}, \quad I_0 = pE \sin \frac{\varphi}{2}, \qquad (1)$$

every subsequent term of which, beginning with the second, is determined in descendent order of accuracy.

At the base of the cone (see above) $\delta = \frac{\pi - \varphi}{2\pi} \sim \frac{1}{2}$, therefore

$$I_A \approx E \frac{p \sin \frac{\varphi}{2}}{1 - \frac{1}{2} p}. \qquad (2)$$

At the vertex $\delta \approx \cos \frac{\varphi}{2}$, therefore

$$I_M \approx E \frac{p \sin \frac{\varphi}{2}}{1 - p \cos \frac{\varphi}{2}}. \qquad (3)$$

In the case of an even screen the brightness

$$I_\perp = pE. \qquad (4)$$

From (2) and (4) it follows that near the edges of the cone there is always $I \ll I_\perp$. From (3) we obtain that p being known, I_M assumes its maximum values with $\cos \frac{\varphi}{2} = p$** (i. e. $\varphi \to 0$ with $p \to 1$). By substituting it into (3) we obtain $\max I_M \approx E \frac{p}{\sqrt{1 - p^2}}$. If $p = 1$, then $\max I_M = \infty$. However, practically the best $p \approx 0.95$ (thereby $\varphi \approx 35°$). Hence the brigthness of the middle of the cone will exceed that of an even screen no more than thrice. On the average, however, $I \approx I_\perp$, and therefore, in the case of illumination with visible light, screens covered with cone-like or any other depressions offer no particular advantages as compared to even ones (see dotted curves in Fig. 2***)

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* As it is obvious that $I_{iM} < \bar{I}_i < I_{iA}$, the value obtained by this method for $I_{(i+1)M}$ will necessarily be underestimated, while that for $I_{(i+1)A}$ will be overestimated.

** The quantity of φ, with which the mean value of I for the cone is a maximum one, will be but little different.

*** The dotted curves are plotted on a scale different from the rest.

The radiation of a cone drops sharply if observed at angles exceeding $\frac{\varphi}{2}$, when the brightest parts of the cone are being screened by its edges. It may thus be seen that the cone acts like a system directing the light flow.

2. Unlike the case just considered, of certain practical interest are self-luminescent uneven screens coated with fluorescent material of constant or variable action. We know, for instance, that screens of this kind are being applied for various purposes at the State Optical Institute.

The problem can be reduced to the preceding one, Lambert's law holding fairly well for powdery phosphors.

For the total brightness I we obtain again formula (1), where I_0 is the initial surface brightness of the fluorescent layer, consequently depends neither on E nor on φ. In accordance herewith, instead of (2) and (3) we find that near the base of the cone

$$I_M \approx I_0 \frac{1}{1-\frac{1}{2}p}, \tag{5}$$

whereas near the vertex

$$I_A \approx I_0 \frac{1}{1-p\cos\frac{\varphi}{2}}. \tag{6}$$

From (5) it follows that with p approaching 1, we always have $I_M \approx 2I_0$, i. e. about twice as great as for an even screen. According to (6), I_A increases as φ becomes smaller. With $p \approx 0.95$ and $\varphi = 35°$, $I_A \approx 10\, I_0$; in the limit case with $\varphi = 0$, $I_A \approx 20\, I_0$. The mean brightness of a surface with cone-like depressions may, consequently, be many times that of an even screen. With p sufficiently great, the relation between the light flows from a cone-like screen and from an even screen, equivalent to the base of the cone, is approximately equal to the relation between the surface of the cone and the area of its base.

3. Of greatest interest seem to be those cases when the screen is manufactured from a phosphor luminescent under the action of «red» light (flash).

The distribution of the «red» light intensity, I', after its being repeatedly scattered inside of the cone, is analogous to the distribution of visible light for case 1, and may be expressed by formulae (1), (2) and (3) (for convenience sake the coefficient of scattering of the «red» light is denoted by p'). The flash I_0 is proportional to I' and to the absorption coefficient of the «red» light $q' = 1 - p'$

$$I_0 = \eta q' I' \approx \eta q' I'_0 \frac{\sin\frac{\varphi}{2}}{1-p'\delta}, \tag{7}$$

where η is the coefficient of proportionality.

Unlike the preceding case, the initial distribution of visible luminescence over the cone is here no regular one. Let us apply again the same procedure of rough estimation, assuming the intensity of light after an $i+1$-th scattering I_{i+1} to be proportional to the intensity after an i-th scattering I_i, to the scattering coefficient p and to the solid angle δ at which the luminescent surface of the cone is being viewed. Summing up a series of magnitudes of I_i, we find with the aid of (1) and (7) that

$$I \approx \eta q' I'_0 \frac{\sin\frac{\varphi}{2}}{(1-p'\delta)(1-p\delta)}. \tag{8}$$

For a flat screen

$$I_\perp = \eta q' I'_0. \tag{9}$$

From (8) and (9) we obtain that the efficiency of the cone $\left(Q = \frac{I}{I_\perp}\right)$,

i. e. Q, increases with $p \to 1$:

$$Q \approx \frac{\sin \frac{\varphi}{2}}{(1-p'\delta)(1-p\delta)}. \qquad (10)$$

If we put $p' \approx p \approx 0.95$ *, then Q takes on its maximum value with $\varphi \approx 20°$. The brightness of the screen near the edges $\left(\delta = \frac{1}{2}\right)$ will then make up ~ 0.5 of the brightness of a flat screen. It increases rapidly towards the vertex, near which $Q \approx 40$.

The mean brigthness of a screen with cone-like depressions may thus many times exceed that of an even screen. The cones serve as a kind of trap for the «red» light.

Similar to the preceding case; the radiation is directed within a narrow solid angle. The store of the light sum of a screen with cone-like depressions is nearly as many times larger than the store of a corresponding even (flat) screen, as many times the lateral surface of the cone is larger than the area of its base.

Cone-like screens with a self-luminescent surface can be made use of as a source of light of controlled action.

The effect of cone-like screens being independent of their linear dimensions, a single cone may be replaced by a number of similar cones of smaller size, i. e. by a practically plane screen of a honey-comb (cellular) structure. In order to make the cells tightly adhere to one another leaving no gaps, it is more convenient to make them pyramidal instead of cone-like.

We have manufactured such a honey-comb structured screen; in which $\varphi \approx 20°$, $p' \approx p \approx 0.85$. Its mean brightness has been found to be about three times that of a plane screen, which agrees fairly well with the values of $Q_M \approx 0.5$ and $Q_A \approx 6$, as obtained from (10).

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* The reflection factor of the «red» light is also vary large.

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TECHNICAL PHYSICS

A SCATTERED SYSTEM OF RADIATORS IN SOUND REPRODUCTION FOR MOTION PICTURES*

By L. D. ROSENBERG and B. D. TARTAKOVSKY

(Communicated by S. I. Wawilow, Member of the Academy, 21. VI. 1943)

1. Experiments were performed using the apparatus described in our previous note (1). The motion pictures were demonstrated on a screen 2×3 m hanging in the front of the hall. Behind the screen there was placed a compact group of sound sources of the type used in scattered system, thus imitating a concentrated system usual in motion picture demonstrations. Both systems had identical quality characteristics.

2. Fragments of films containing various kinds of sound material: a monologue, a dialogue, solo singing, chorus, fragments from chamber and orchestra music etc. were used for tests. These fragments were offered to the public without any previous warning. The listeners were given question-cards and each was to fill out his card without consulting others.

3. Two tests were performed. In the first, the film fragments were demonstrated alternatingly by means of a concentrated and a scattered system, and the listeners were asked whether they were aware of any difference. Nearly a half did not notice any difference at all; about 35 per cent noticed some difference with respect to light intensity, trembling of the image, loudness of the sound, etc., while only about 15 per cent noticed that something happened with the sound. However, these were only rare instances when there was a clear foundation of just exactly what did take place.

In the second test the same demonstration was repeated, but the attention of the audience had been previously fixed on the character of sound. The listeners were asked to tell whether there was any difference in the character of sound, and if that was the case, were asked to tell what seems to produce a more natural impression. About 80 per cent of the listeners noticed the difference, with 60 per cent of them rating higher the sound of the scattered system. 20 per cent of the listeners noticed no difference at all. The high percentage of those who noticed the difference led us to the idea that some listeners, because of the warning, were apt to treat with exaggerated suspicion any casual changes of sound. For that reason, a checking-up experiment was performed; after a warning the audience in the same way as in the basic experiment, one fragment was demonstrated twice in identical conditions (by means of the scattered system). About a half of the

* The investigations described here had been performed by the authors in January 1941, but were not published up to the present for some technical reasons. After completion of the work a communication was published in American press that in USA a combined system of concentrated and scattered loudspeakers has been devised under the name of «Fantasound», this system being applied in one of the American theaters. This apparatus was devised specially for the reproduction of the picture «Fantasy» by Disney, the famous American producer.

listeners said that they noticed a difference. Thus, we may consider that the original 80 per cent of noticeability is an exaggeration.

4. The tests have shown that the audience that had not been warned scarcely noticed any difference between the sound reproduction of a concentrated and a scattered system. Only after a warning following a second demonstration did the listeners notice any difference. Quantitative results of the tests show that the scattered system for sound reproduction is no less, and perhaps even somewhat more satisfactory than the concentrated one*. Hence it follows that the scattered system may be successfully applied in moving picture theaters. As has been shown in ([1]), it is particularly efficient in the halls shaped unfavourably from the viewpoint of acoustics, and where the application of the concentrated system encounters very great, sometimes insurmountable difficulties.

5. The fact established above, of the attachment of the sound image having no independent localization to the screen image, on the one hand, and the sound effect of the sounding of the hall volume, on the other, have suggested the idea that it is possible to apply the combination of the scattered and the concentrated systems for obtaining the quasistereophonic effect. Technically this may be achieved by the change of relations of sound power radiated by each of these systems, in accordance with the content of the picture, the total level of sound intensity remaining constant.

The complex effect may be achieved in the sound synchronization of a stereoscopic film. But in the case of usual films, we may also expect a certain degree of stereophonism.

A special test has been performed for checking up this supposition. Fragments from films have been demonstrated in one case when using the concentrated system, in another case, when combining the scattered and the concentrated system.

The ratio of the power of the two systems was altered by the operator according to a previously formed plan in accordance with the character of the visual image. 90 per cent of the listeners were aware of the difference in sound, and with a few rare exceptions admitted the superiority of the combined system of sound reproduction.

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16. VI. 1943.

REFERENCES

[1] L. D. Rosenberg and B. D. Tartakovsky, C. R. Acad. Sci. URSS, XXXI, 883 (1941).

* This conclusion, which at first may seems strange, can be explained by the fact that the sound image which has no definite localization (in the case of the operation of the scattered system) becomes easily attached not only to a living visual image, as has been shown in the previous work, but to the image on the screen as well and follows it. The separation of the sound image from the living image takes place only in the case when a warned listener consciously fixes his attention on it.

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CHEMISTRY

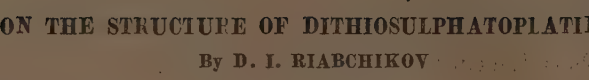
ON THE STRUCTURE OF DITHIOSULPHATOPLATINITE

By D. I. RIABCHIKOV

(Communicated by A. E. Arbusov, Member of the Academy, 2. V. 1943)

The compound ($Pt : 2S_2O_3^=$) obtained by me from the reaction of sodium thiosulphate with chloroplatinite has proved to consist of two forms of different solubility (1). According to the analysis data, the two forms correspond in composition to the formula $K_2[Pt(S_2O_3)_2]$.

On the theory of Werner, a compound of this composition may be supposed to exist in two isomeric forms:



cis-isomer trans-isomer

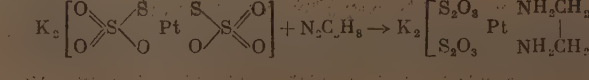
Yakshin (2) has shown that the two compounds of different solubility have also an essentially different molecular conductivity. Moreover, the more easily soluble form exhibits a considerable variation of μ as the solution is diluted. This phenomenon is undoubtedly connected with the hydration of the compound, which appears from the experimental data to proceed at a higher rate in the case of the easily soluble form than in that of the poorly soluble.

It is essential, however, to establish the nature of the isomery by direct determinations. The thioureal reaction of N. S. Kurnakov, generally employed for that purpose, did not lead here to the desired result, for the same compound $[Pt\,3SCN_2H_4 \cdot S_2O_3]$ was formed in both cases.

It was found that the problem can be solved by using ethylenediamine with which the two forms of $K_2[Pt(S_2O_3)_2]$ give compounds of different composition. The ethylenediamine molecule adds to the central atom of the complex by a closed cycle and necessarily occupies two coordination places drawn together.

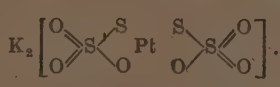
The ethylenediamine was added to a heated solution of the easily soluble form. No visible changes took place and the solution remained perfectly clear. When a solution of the salt $[Pt\,4SCN_2H_4]Cl_2$ was added to it, there was at once observed a precipitation of yellow flakes. Because of being left for a long while in the mother liquor, these flakes disintegrated into powder.

According to the data of analysis, the product thus formed corresponded to the formula $[Pt\,N_2C_2H_8(S_2O_3)_2][Pt\,4SCN_2H_4]$. From that it may be inferred that its anion $[Pt\,N_2C_2H_8\,(S_2O_3)_2]^=$ may possibly have been the result of the addition of a molecule of ethylenediamine to the initial form of the dithiosulphate, which proceeded as follows:



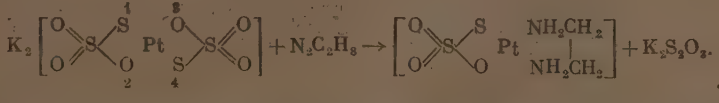
The ethylenediamine molecule takes the place of two oxygen atoms of the thiosulphate residues. The thiosulphate residues open their cycles and, being now held only by means of sulphur atoms, occupy one coordination place each.

The very fact of the ethylenediamine molecule finding a place within the inner sphere of the complex is evidence that the identical (O and O) coordination substituents of the compound in question were in cis-position. On that ground the easily soluble form may be taken to have the following structure:



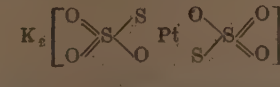
Quite different is the action of the ethylenediamine upon the more sparingly soluble form. When a hot solution of the latter is added with ethylenediamine, a light-yellow powder is observed to precipitate immediately. This precipitate is very hard to dissolve. Its composition according to analysis data is $[Pt\,N_2C_2\,H_8 \cdot S_2O_3]$.

In this case the reaction may be taken to proceed as follows:

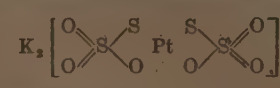


The intramolecular rearrangement within the thiosulphate complex goes through the following stages. At first the ethylenediamine molecule comes with one of its ends to replace one of the oxygen atoms (say, 3). The atom of sulphur (4) in the thiosulphate group with the open cycle, being weakened now by its trans-partner, is torn off from the central atom under the influence of the ethylenediamine molecule closing its cycle. In that way the ethylenediamine molecule displaces two dissimilar (O and S) coordination substituents.

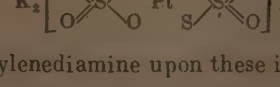
It follows accordingly that the less easily soluble form should be ascribed a trans-structure, viz:



Conclusions. 1. The two forms of the compound $K_2[Pt(S_2O_3)_2]$, differing in solubility and molecular conductivity, are stereoisomers. Of them the more easily soluble form should be assigned a cis-structure



while the other form is a trans-structure



2. By the action of ethylenediamine upon these isomeric forms the compounds

$[PtN_2C_2H_8 \cdot S_2O_3]$ and $[PtN_2C_2H_8(S_2O_3)_2]$ $[Pt4SCN_2H_4]$

have been obtained.

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REFERENCES

[1] D. I. Riabchikov, C. R. Acad. Sci. URSS, XXVII, No. 4 (1940).
[2] M. M. Yakshin, ibid., XXVIII, No. 4 (1940).

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CHEMISTRY

PRODUCTION OF MAGNESIUM OXIDE FROM DOLOMITE AND ITS USE IN THE MANUFACTURE OF REFRACTORIES

By P. P. BUDNIKOV, Corresponding Member of the Academy, and I. I. RIVLIN

The methods heretofore applied for obtaining magnesium oxide from dolomite are involved and expensive. Among those in use the treatment of calcined dolomite with bicarbonate alkalies and carbonic acid may be mentioned as an instance. Calcined dolomite is also treated with sulphuric acid and sulphates, with nitric acid and nitrates, with hydrochloric acid and chlorides [1–6].

The method suggested below by the authors is to treat calcined dolomite with a solution of ammonium chloride. Our investigation was planned to establish the calcining temperature of dolomite, the concentration of NH_4Cl and the temperature of the reaction in which dolomite is decomposed by a solution of ammonium chloride. The chemical composition (in per cent) of the initial dolomite (raw) was as follows:

SiO_2	1.46	MgO	20.52
R_2O_3	0.20	Loss by calcining	46.90
CaO	31.54		

The dolomite was calcined in an electric crucible at different temperatures for an hour. Then it was finely ground. A weighed portion of it was treated with solutions of ammonium chloride of varying concentration at ordinary temperature, at 60° C and at boiling temperature. The products of the reaction of dolomite with NH_4Cl were analysed for CaO and MgO. The respective temperatures were measured in all tests. In Tables 1, 2 and 3 the results of these tests are given.

Table 1

Calcining $t°$ of dolomite, °C	Precipitate in %		Filtrate in %	
	CaO	MgO	CaO	MgO
800	30.76	17.06	28.0	29.0
900	9.60	9.1	49.0	21.4
1 000	8.64	19.63	47.44	18.57
1 100	6.68	21.43	49.80	16.81
1 200	7.72	34.1	50.56	3.84

Notes. 1. Dolomite was treated with NH_4Cl solution at ordinary temperature. 2. Solution of NH_4Cl is a saturated one.

From this table it is seen that the amount of magnesium oxide in the precipitate increases with the calcining temperature and is a maximum at 1200° C. The

content of MgO in the filtrate decreases accordingly. At 1200° C as much as 90 per cent of the dolomite's content of MgO passes into precipitate and only 10 per cent into filtrate.

The decrease in the amount of MgO contained in the solution in the form of $MgCl_2$ is controlled by the calcining temperature. This is because the increase of the calcining temperature leads to an increase of the amount of periclase which is more hard to react with NH_4Cl than the amorphous oxide of magnesium. Our experiment has shown that an ordinary preparation of MgO is almost completely dissolved after one hour treatment with a solution of ammonium chloride. Only about 1 per cent of its initial weight remains in the precipitate. And if MgO is in the form of periclase, 28.15 per cent of it is left behind under equal conditions.

Table 2 shows the yield of magnesium oxide as related to the temperature at which calcined dolomite is treated with ammonium chloride solution.

Table 2

t° of treatment with NH_4Cl	Precipitate in %		Filtrate in %	
	CaO	MgO	CaO	MgO
Ordinary .	7.72	34.1	50.56	3.84
60° C . . .	3.04	9.37	55.79	28.40
Boiling . .	2.81	6.55	55.83	31.73

Note. Calcining temperature, 1200° C.

As is seen from Table 2, the best results are obtained at 15 — 18° C. At higher temperatures more MgO remains in solution.

In Table 3 the yield of MgO is given with reference to the concentration of NH_4Cl.

Table 3

Concentration of NH_4Cl in water solution in%	Precipitate in %		Filtrate in %	
	CaO	MgO	CaO	MgO
10	3.06	31.0	55.80	7.16
18	4.66	32.51	53.92	5.83
Saturated .	7.72	34.12	50.56	3.84

Note. Calcining temperature, 1200° C.

It may be inferred from these data that the concentration of NH_4Cl is a less powerful factor in determining the yield than is the calcining temperature or the temperature of treatment with ammonium chloride. The fluctuations in the distribution of CaO and MgO do not go beyond 3—4 per cent.

The magnesium oxide thus obtained can be used in the manufacturing of magnesite and forsterite refractories. Forsterite refractories consist mainly of magnesium orthosilicate (Mg_2SiO_4) and have a considerable advantage over silica brick (dinas) in being of basic chemical character and melting at a higher temperature.

In the system MgO—SiO_2 forsterite ($2MgO \cdot SiO_2$) is the only stable compound at above 1562° C. Therefore no refractory material but consisting mainly of forsterite can possibly be obtained from magnesium silicate (7). This consideration has determined the direction of our research.

We synthesized samplzes of forsterite refractory material from serpentinite and magnesium oxide obtained from dolomite.

The production of synthetic forsterite is based on the following reactions. First a thermal decomposition of serpentinite takes place:

$$3MgO \cdot 2SiO_2 \cdot 2H_2O = Mg_2SiO_4 + MgSiO_3 + 2H_2O$$

with the formation of a mixture of ortho- and meta-silicates of magnesium. The meta-silicate (clinoenstatite) melts at 1557° C, the ortho-silicate (forsterite) at 1890° C.

The next stage is the reaction between the meta-silicate and the oxide of magnesium:

$$MgSiO_3 + MgO = Mg_2SiO_4.$$

The serpentine we used had the following composition in per cent: SiO_2, 38.30; Al_2O_3, 0.86; Fe_2O_3, 7.25; FeO, 1.07; Cr_2O_3, 0.33; MnO, 0.15; CaO, traces; MgO, 38.66; H_2O, 12.74; SO_2, traces; $K_2O + Na_2O$, 0.27.

Of a mixture of this serpentinite with MgO we made three different samples: No. 1, with 25 per cent of serpentinite and 75 per cent of a solid residue containing 90 per cent of MgO; Nos. 2 and 3, with 30 and 40 per cent of serpentinite, respectively. The mixing and moulding were made by the wet method. The moulded samples were calcined in a cryptol oven at 1580° C. Both the heating and cooling of the oven were conducted at a slow rate. The samples were kept at the just mentioned temperature for 20 minutes. The result of the calcining was a dense well caked refractory.

Conclusions: 1. The possibility of separation of magnesium oxide from dolomite has been proved. There has also been established the optimum calcining temperature (1200° C) for dolomite and the temperature of its treatment with ammonium chloride (15—18°).

2. Out of a mixture of dressed dolomite and serpentinite samples of forsterite refractory have been prepared.

3. This method can be made valuable economically if the mother liquor of soda plants, which contained ammonium chloride, is used for the treatment of dolomite and if ammonia is returned to the cycle of the soda production process.

Academy of Sciences
of the Ukrainian SSR.

Received
28. VIII. 1943.

REFERENCES

[1] K. Kippenberger, ZS. f. anorg. Ch., **6**, 177 (1894). [2] Treadwell u. Reuter, ibid., **17**, 170 (1898). [3] Knorre, ibid., **34**, 260 (1903). [4] Auerbach, ZS. Elektroch., **10**, 161 (1904). [5] Ullmann, Enzykl. der techn. Ch., 1 Aufl., **7**, 678—681. [6] V. M. Goldschmidt and R. Kundsen, J. Amer. Cer. Soc., **19**, 93 (1936). [7] А. С. Бережной, Сб. работ Ин-та огнеупоров,, вып. XLVI, **87**, Харьков (1941).

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GEOCHEMISTRY

ON THE VARIABILITY OF GASES OF THE SOUTHERN ISHIMBAY MASSIF

By G. D. HALPERN

(Communicated by S. S. Nametkin, Member of the Academy, 27. V. 1943)

Analyses of gases from wells sunk at the Southern Ishimbay massif, which had been carried out according to our instructions by A. M. Protasova, revealed a relatively stable gas composition of some of these wells (airless gas). At the same time the composition and properties of the gases show a definite zonality similar to that which has earlier been reported for oils (1) *. From 8.4 per cent at the centre of the deposit the concentration of hydrogen sulphide goes down to 2 per cent at the periphery; the concentration of mercaptane falls from 0.6 to 0.05 per cent, and that of carbon dioxide from 1.2 to 3 per cent. The specific weight of the hydrocarbon part of the gas, calculated after methane taken as a unit, goes down from 2.53 to 1.42. This should be taken to mean that the concentrations of ethane, butane and propane are decreasing in the same direction as in hydrogen sulphide.

Unlike this the concentration of methane rises from the centre of the deposit towards its periphery where the «poorest, dry» gases are to be observed.

Thus, the mode of distribution of gases throughout the Southern massif goes fully to confirm the statement as to the poor communication between the separate areas of the deposit, which has been advanced by the present author with respect to the Ishimbay oils. The «gas-caps» of the Caucasian oil deposits are distinguished by a much higher stability of composition.

This does by no means signify that the conditions of generation and metamorphization of oil that prevail in the Caucasian sandy reservoirs are essentially different from those observed in a reefogenic bed. Similar to the latter, a number of most important factors acting upon the oil are of zonal character here, too.

Because of the ease with which oil and gas migrate throughout the thickness of the bed, their peculiar characters become less pronounced, masked, whereas in a reefogenic deposit where there is no appreciable mixing either of oil or of gases, these characters remain clear cut.

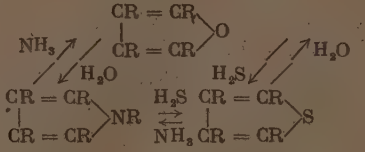
In our analysis of the Ishimbay oils we pointed out the reducing conditions prevailing in the central zone of the deposit. Data now available on the nature of gases make it possible to give a more explicit definition of the essence of these conditions. The maximum generation of hydrogen sulphide corresponds to the core of the rock.

To our opinion, it is due to biogenic reduction of the sulphates whose intensity is regularly decreasing towards the periphery of the deposit. It should be noted here that the concentration of free hydrogen in the gases of the Southern massif is rather low and does usually not exceed 1—2 per cent.

* Tabulated data and maps see in (1,2).

The fact that the maximum concentrations of mercaptane usually coincide with those of hydrogen sulphide, points to the concomitance of their generation processes.

We are inclined to believe that both the higher sulphurous compounds which are so typical of the Ishimbay oils, and the mercaptane are products of interaction between hydrogen sulphide and the non-hydrocarbonic components of oil. Metathetic reactions involving hydroxide, the amino-group, furfurane and nitrogen are very likely to occur, in particular, pyrrolic compounds, proceeding according to Yuriev's scheme. (2):



There should evidently exist some enzymic mechanisms that facilitate biogenic transformation according to this scheme, which has until now been reproduced only by using comparatively rough catalytic methods.

According to this scheme, as a source of mercaptane may serve alcohols and amines of the parent material. The limited concentration of the lower non-hydrocarbonic components in oils seems to us to account for the limited concentration of the lower sulphur compounds even in oils high in sulphur. If they owed their origin to hydrocarbons, as has been believed by many investigators, then, having for centuries been acted upon by hydrogen sulphide, the concentrations of sulphur compounds should many times exceed those actually observed.

Hydrogen sulphide and carbon dioxide are usually considered as antagonists. The more noteworthy is the occurrence of the maximum carbon dioxide concentrations at the zones of a maximum generation of hydrogen sulphide (the low absolute values of both being taken into consideration).

The low alkalinity of the oil-waters of Ishimbay, as well as the absence in the rocks of copper, nickel or iron carbonates, seem to be evidence against the possibility of an inorganic origin of CO_2, at the expense of interaction with hydrogen sulphide.

However, from a comparison of the concentration zones of the two gases it is quite natural to conclude that the processes that lead to the formation of carbon dioxide and hydrogen sulphide are concomitant. Carbon dioxide is probably a by-product of the life activity of those micro-organisms which are responsible for the transformation of sulphates into hydrogen sulphide.

It is more difficult to gain an idea of those processes which lead to the formation of the closest homologues of methane in the central zones of the deposit. At any rate it is here anything but polymerization of methane, which many writers were inclined to regard as the main process leading to oil generation.

The zones of maximum development of polymerization processes correspond, as already pointed out in an earlier note (1), with the peripheral areas of the deposit rather than with its central part. It is in the peripheral areas that the maximum concentrations of methane and minimum concentrations of its gaseous homologues are to be observed.

For those who delight in hydrogenation hypotheses of any kind, we avail ourselves of this opportunity to point out that our selective analyses for the Southern and Buranchinsky massifs gave a content of free hydrogen not above 2 per cent.

The sharp difference in the specific weights of gases from the peripheral and the central zones of the deposit, which is observable under the conditions of the Southern massif, makes us regard the processes that lead to the formation of methane as being antagonistic to those responsible for the formation of ethane, propane and butane.

The continuity of the transition from methane to its very high homologues, which has been postulated for oil paraffin hydrocarbons by a number of students,

is refuted by field evidence. The formation of methane is no a link in the continuous chain of oil generation, but only an independent, although often concomitant process.

This process shows its optimal development in zones where the formation of lower homologues is attended with difficulties. On the other hand, zones, where the formation of methane is clearly suppressed, are distinguished by an increased formation of ethane, propane and butane.

The idea suggests itself, that these homologues of methane may possibly present nothing else than anomalous products of the same process which under different conditions would lead to the formation of pure methane, and whose nature still remains obscure.

If we were to consider deposits of light oils with the «rich, wet» gases associated with them, and do this from a roughly quantitative point of view, with this, we might easily be misled and adopt a continuous, presumably genetical series from methane to hard paraffins.

The occurrence of «poor, dry» gases in deposits of heavy oils clearly bears witness to the existence of an organic break in this series, more strictly speaking, to the existence instead of a single series, of two independent series at least: of one for methane and its closest homologues, and another, for the hard and liquid paraffin hydrocarbons of oil. The subject is to be treated elsewhere.

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REFERENCES

[1] G. D. Halpern, C. R. Acad. Sci. URSS, XLI, No. 4 (1943); Bull. Acad. Sci. URSS, sér. techn., No. 5—6 (1943). [2] Ю. К. Юрьев, ЖОХ, 6, 972, 1669 (1936). [3] G. D. Halpern, Bull. Acad Sci. URSS, sér. techn., No. 8 (1943).

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GEOCHEMISTRY

ON THE GENETIC PECULIARITIES OF THE MANGANESE ORES OF THE POLUNOCHNY DEPOSIT

By A. L. ZAGHIANSKY

(Communicated by V. G. Chlopin, Member of the Academy, 1. VI. 1943)

In connexion with the prospecting work carried on at and operation of the Polunochny manganese ore deposit, the author was able to collect new data necessary for solving a number of important problems relating to the geology of the manganese deposits of the North Urals and their evaluation.

Until recently the opinion was held that the oxide («oxidized») ores of the Polunochny deposit had been formed as a result of the process of oxidation of manganese carbonates and migration from carbonate rocks chiefly of alkaline-earth components, silica and sulphur (1).

The material collected show that the formation of the greater part of the oxide ores of the Polunochny deposit is associated with a process of primary deposition of manganese oxides (primary sedimentary ores). The conditions of deposition of these ores in the area of the Polunochny deposit have much in common with the formation of the Nikopol and Chiatury manganese ores.

The primary manganese oxide ores are concentrated in the western part of the deposit, while the eastern portion of the mineralized area is characterized chiefly by carbonate and siliceous (opal) carbonate ores, with the exception of some areas of redeposited oxide ores.

The development of oxide ores in the western portion of that part of the former shelf which was nearest to the shore, and the transition of these ores according as they recede eastward from the shore into carbonate ores (while still farther east they pass into opal-carbonate manganese ores), are here the specific features of the zoning of the mineral-forming processes.

The main ore-bearing stratum of the western part of the deposit is characterized by an alternation of interbeds (3—8 mm thick) mostly consisting of wad with interbeds composed mainly of minerals of the kaolin group, fine-grained quartz, and to a lesser degree, of wad and glauconite (the thickness of these strata attains 8—10 m). An interesting feature is the presence within these beds of lenses and lenticular intercalations (up to 1.2 m thick) of argillo-arenaceous rocks with concretions attaining 8 cm in diameter and consisting in the main of pyrolusite. The origin of these sediments, for the greater part coarse-grained, with manganese concretions, may be related to the agency of sea currents.

In the northern part of the exploited area of shaft No. 3, over the stratified ores there occur patches of compact clays with friable concretions consisting mainly of wad. Chiefly above the horizon of stratified ores there are patches of argillo-arenaceous and areno-argillaceous sediments with manganiferous formations, not infrequently passing into the so-called lumpy porous ores (the pyrolusite content in the ore portion attaining 60—70 per cent).

The formation of ore concentrations in which higher manganese oxides are predominant (patches with pyrolusite concretions, lumpy porous ores, etc.) is in the main related to the phenomenon of dehydration of the minerals of the psilomelane-wad group which prevailed in the primary sediments (this does not exclude oxidation and subsequent dehydration of the carbonate minerals that found their way to the weathering zone, which evidently occurred in some places).

The chemical differentiation in the eastern part of the shelf, relatively more remote from the shore, tended toward the formation of deposits with the development of mangano-dolomite oligonite, calcite-rodochrosite. Unlike the western

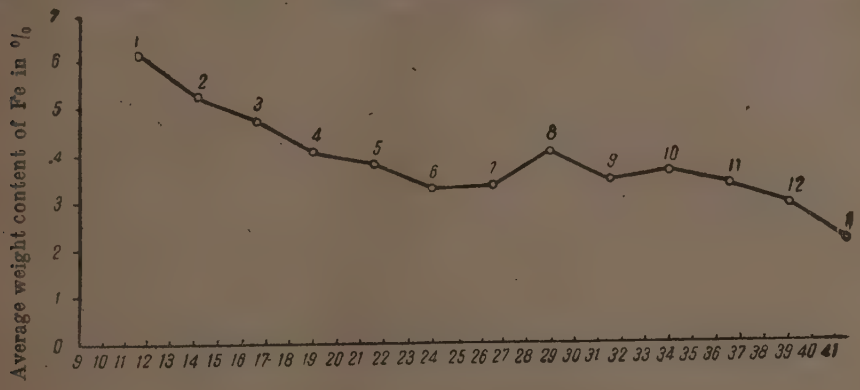

Fig. 1

ultra-oxidation facies: under conditions of a reducing medium, there were formed opal-carbonate sediments with a considerable development of oligonite. The latter are also characterized by the presence of pyrite and barite.

Important data, shedding some light on the origin of oxide manganese ores of the Polunochny deposit, are furnished by the geochemical peculiarities in the distribution of iron in manganese deposits. From the plot showing the ratio of manganese and iron in the manganese ores of the Polunochny deposit (Fig. 1) it may be inferred that with an increase of the manganese content, the content of iron decreases. In the ores with a lower manganese content, the content of iron is relatively high.

If we were to adopt the viewpoint that oxide ores have been formed as a result of oxidation of carbonate ores, the relation between these components should be a direct one, since along with an enrichment of carbonate rocks with manganese, an enrichment in iron should also take place.

The diagram presenting the relation between the content of the manganese and phosphorus * (Fig. 2) shows that the rich oxide ores deposited in the shallow-water part of the shelf are rather poor in phosphorus. The highest phosphorus concentration is associated with areas of poor, mainly siliceous-carbonate rocks formed in the deep-water part of the shelf.

These data are also of a considerable practical importance. They show a comparatively low ratio of phosphorus to manganese in oxide ores (lower than 0.005), which along with their increased manganese content is the main reason why they are considered as a valuable raw material for ferro-manganese.

Considering the concentrations of oxide manganese ores in the region of the Polunochny deposit as an area of a comparatively shallow-water part of the shelf from the North-Ural manganese basin, it is quite possible that new areas of develop-

* In preparing the diagrams (Figs. 1 and 2) the weighed mean content of the individual components was determined (from some hundreds of samples) for ranges of 2.5 per cent manganese. To every point of the diagram correspond 10 to 20 analyses.

ment of these rocks north and south of the Polunochny deposit will be discovered along the contact of Palaeozoic rocks with Mesocenozoic strata. Owing to the fact that a considerable part of the zone of this contact is cut by the systems of the Lozva and Sosva rivers, large areas of primary oxide ores have been eroded (the manganese ores in these areas are frequently represented by ores of deeper horizons, such as e. g. the phosphorous ores of the Marsyat deposit). As a result, in the western

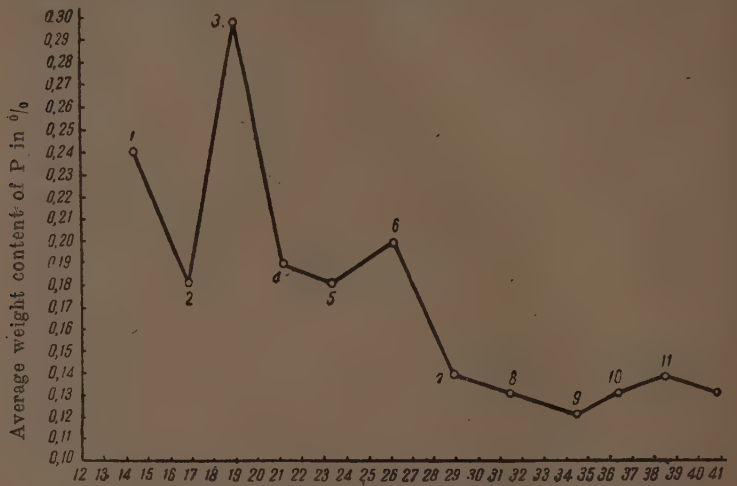

Fig. 2

part of the Polunochny deposit there is a considerable area of alluvial oxide ores (in the region of shaft No. 2, the thickness of the latter attains 5 m). It is of interest to note that hole No. 24a which penetrated about 15 m of alluvial ores at a depth of 76 m, struck against Quaternary brown clays with poorly waterworn pebbles of oxide ores.

The search for primary deposits of oxide ores, the principal raw material of ferro-manganese, should be conducted in the Northern Urals in the closest proximity of the contact of Palaeozoic rocks with Tertiary deposits, where it recedes to a considerable distance from the recent river valleys.

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Received.
3. I. 1943.

REFERENCES

[1] Е. П. Молдаванцев, Труды Конфер. по генезису железа, марганца и алюминия, изд. АН СССР, 1937.

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PLANT PHYSIOLOGY

EFFECT OF REPEATED DEHYDRATION ON PHOTOSYNTHESIS

By V. A. BRILLIANT

(Communicated by L. A. Orbeli, Member of the Academy, 19. VI. 1943)

There is an accumulation of evidence in the literature to the effect that the withdrawal of water from the assimilating tissue of plants has a decisive influence upon photosynthesis not merely at the time of exposure, but also in the later period, because certain changes have been brought in the physiological condition of the cell and in its photosynthetic apparatus. Experiments by Walter (5), Stalfelt (4) and Kaltwasser (3) on aquatic plants and mosses have shown, for instance, that watered plants that have suffered dehydration continue to exhibit a reduced rate of photosynthesis for some time after, and that the restoration of the normal rate depends upon how long they were allowed to go without water.

It appears certain from the available data that the water regime of a plant is as an essential part of its history as the light and heat regimes, for a change in the water store of the assimilating tissue has a definite after-effect on the rate of photosynthesis, which will not disappear until long after the normal water supply has been restored.

The question of the stability of this after-effect or, in other words, of the reversibility of the alterations of photosynthetic activity caused by dehydration, is of great interest from the agricultural point of view, since in the case of a plant that has suffered drought the return to normal life-activity very favourably affects the yield both quantitatively and qualitatively.

Closely related to this question is that of the dependence of photosynthesis upon a reiterated drying of the assimilating tissue. It has been dealt with by me in a cycle of experiments on the leaves of cotton (*Gossypium hirsutum*). The plants were grown in pots at a constant soil moisture. By means of the eudiometric method the assimilating capacity of the leaf, or the potential rate of photosynthesis, was determined under a high concentration of carbon dioxide (about 4 per cent) and artificial illumination (a bulb of 300 W placed at a distance of 20 cm). The content of CO_2 in the gas mixture before and after the exposure was estimated with the Bonnier and Mongen apparatus.

The experiments were run as follows. From a cotton leaf equal fragments, 6 cm^2 in area, were cut out symmetrical with respect to the mid-rib. One of the fragments was dried in the air, the other served as control and was kept in a glass on humid filter paper. After the decrease in weight of the test-fragment was determined, the rate of photosynthesis was measured, and both the test and the control fragments were placed in water to remain there till next day (sometimes for a shorter period). Afterwards the test-fragments were dried for the second time, and the rate of photosynthesis was measured again.

Second dehydration compared to first	Nos.	First drying: Loss of water, % in raw weight	First drying: Photosynthesis: cm^3 of CO_2 per hour per 10 cm^2: test	First drying: Photosynthesis: cm^3 of CO_2 per hour per 10 cm^2: control	First drying: Photosynthesis: Test to control, %	Second drying: Duration of soaking in hrs	Second drying: Loss of water, % in raw weight	Second drying: Photosynthesis: cm^3 of CO_2 per hour per 10 cm^2: test	Second drying: Photosynthesis: cm^3 of CO_2 per hour per 10 cm^2: control	Second drying: Photosynthesis: Test to control, %	Photosynthesis (% to control) after second drying compared to first
Less	1	8.3	1.50	1.50	100	19	+3.2	1.40	1.14	122	+
	2	8.7	1.00	1.43	70	19	5.1	0.99	0.90	100	+
	3*	10.6	0.27	0.20	135	17	9.8	0.26	0.25	104	—
	4	12.8	0.76	1.36	56	19	+4.5	1.20	1.20	100	+
	5	13.2	0.78	1.32	59	19	+1.1	0.85	1.35	63	—
	6	13.5	1.42	1.64	87	21	9.0	0.90	0.60	150	+
	7	22.8	0.18	1.20	15	19	20.4	0.90	1.25	72	+
	8	25.4	0.72	0.96	75	19	13.0	0.60	0.93	65	—
	9	26.6	0.79	0.88	90	19	24.6	0.84	1.20	70	—
	10	27.1	0.26	0.65	40	2	23.3	0.23	0.78	30	—
Equal	11	6.6	0.33	0.42	79	23	6.6	0.44	0.31	142	+
	12	15.9	0.70	1.35	50	19	16.2	0.65	0.80	81	+
	13	17.7	0.53	0.79	67	2	17.6	0.55	0.65	85	+
	14	20.9	0.24	0.58	41	2	20.6	0.43	0.55	24	—
More	15	5.2	0.60	0.77	78	1	6.4	0.57	0.55	104	+
	16	5.6	0.85	1.06	80	1	6.5	0.67	0.67	100	+
	17	9.2	0.84	0.96	85	19	13.5	0.41	0.49	22	—
	18	11.5	0.12	0.19	63	17	12.4	0.14	0.12	117	+
	19	13.3	0.65	1.47	44	19	16.4	0.75	1.35	55	+
	20	16.3	0.14	0.33	43	21	29.9	0.19	0.06	311	+
	21	18.1	0.63	1.37	46	21	22.7	0.48	0.84	57	+
	22	20.2	0.11	0.22	50	21	26.0	0.12	0.15	80	+

*Young leaf.

The data obtained are summarized in the adjoined table. They are separated into three different groups according to the loss of water in the second drying as compared to the first. It is seen that in all three groups a reduction in the rate of photosynthesis (indicated by the minus sign in the last column) was observed after second drying, whenever the loss of water after first drying had exceeded 20—23 per cent of the raw weight (tests 8—10 and 14). In the other tests, in which first dehydration had not surpassed this limit, no depression of photosynthesis but in exceptional cases resulted from the second treatment. The exceptional cases are test 3 when a young leaf was experimented upon (according to some observations of mine, young leaves are more sensitive to dehydration, which may account for the result) and test 17 which I find no possible cause to explain away. In the majority of tests (1; 2, 4—7; 11—13, 15; 16, 18—22) the second dehydration whether it had been less or more strong than the first, produced an increase of photosynthetic activity (compared to the control), sometimes even a very considerable one (indicated by the plus sign in the last column).

It is known that the depression of photosynthesis caused by the dehydration of the assimilating tissue is irreversible to a varying degree according to the strength, duration and character of the process of water withdrawal. It is therefore easy to understand why the changes that arise on a sufficiently strong drying of the leaf become able after a new dehydration, even it is less strong than the first has been, to depress the assimilating power of the plant (tests 8—10 and 14).

Although in the other experiments the dehydration was not as strong, it is still difficult to explain the fact that in nearly all cases the rate of photosynthesis after a second drying of the leaf tissue was higher compared to the control than after the first drying, even though more water may have been lost in the former case.

The possible explanation is that the alterations taking place in the cells of a leaf subjected to a not too deep dehydration are of a hardening nature and go to

increase the resistance of the photosynthetic apparatus to further withdrawal of water. Just in what this hardening process consists is very hard to tell, because the inner mechanism of photosynthesis is obscure, in general, and so is also its dependence upon the water factor. The fact revealed by my data [1] is that the dark or protoplasmic reactions of the photosynthesis are the most immediately affected by the withdrawal of water from the assimilating tissue, and this gives us ground to suppose that the increase of the photosynthetic activity after a second dehydration is likewise due to some remodelling of the protoplasmic apparatus that participates in photosynthesis. Provisorily this remodelling may be described as hardening, whereby nothing is definitely said about the process itself, on which only very general considerations can be expressed at this juncture.

Thus, it may be supposed that the colloid-chemical modifications of the protoplasm due to dehydration are reflected in the regulatory activity of the cells, performed through its enzymic system and, in particular, by that complex of enzymes which directly participates in photosynthetic reactions. Certain shifts in the operation of these enzymes as regards their activity and direction are perhaps the responsible factors in the hardening of the photosynthetic apparatus against loss of water.

There is reason to believe that the reversibility of the effects caused by the withdrawal of water from the assimilating tissue, whereby the initial rate of photosynthesis is regained, does not signify by any means that all changes induced by dehydration in the photosynthetic apparatus are annihilated. In some cases the after-effect of the dehydration is not to reduce, but rather to increase the index of photosynthesis, as compared to the control. Apparently, various conditions combine to influence the transition from a poorly to a richly watered state (or the reverse transition), and according to what their combination is, the process of photosynthesis may be inhibited or else stimulated. In this light may perhaps be explained the reaction of the photosynthesis to a second dehydration. It is interesting to point out the solitary case when an increase in the rate of photosynthesis was observed after first dehydration (test 3). This effect to which Danilov[2] gave the name of «Brilliant phenomenon» refers to a young leaf. It is just in this case that the second treatment of the tested leaf fragment caused a reduction of the relative rate of photosynthesis, as compared to the first treatment.

The conclusion to be drawn from these results is that if the action of dehydration on photosynthesis is described as irreversible, this does not simply means that all alterations induced by dehydration are extinguished and the cell returns to its initial physiological state. It is far more likely that while some of the elements of the photosynthetic apparatus which is a part in the complicate dynamics of cellular activity, suffer easily restorable quantitative alterations under the influence of dehydration, others undergo more stable transformations of a qualitative character, and so when some water has been lost or gained, the photosynthetic apparatus as a whole and the entire physiological state of the cell are not what they have been before the change. Therefore when water is withdrawn from the assimilating tissue for a second time, the process is not simply repeated, and though the turgor and even the rate of photosynthesis may have been completely restored, the cells suffering a new dehydration are likely to be in an altered state, to which they have been brought by the first drying. This altered state I conventionally describe as hardening. To disclose the nature of this phenomenon and to state precisely the conditions under which it appears is the object of further investigation.

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Academy of Sciences of the USSR.

Received
24. V. 1943.

REFERENCES

[1] В. А. Браславцев и М. Н. Прелашвили. Труды БИН, сер. IV, Экспер. бот., 5, 88 (1940). [2] А. Н. Данилов, там же. 3, 1 (1938). [3] J. Kaltwasser. Protopl., 29, 498 (1938). [4] M. Stalfelt, Planta, 27, 30 (1937). [5] H. Walter. Protopl., 6, 113 (1929).

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HISTOLOGY

CELL CHANGES IN CONNECTIVE TISSUE DURING REGENERATION. I

By T. M. YAKOVLEVA

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The connective tissue of the fin in amphibiae has a collagenous stroma, its cells have nuclei typical of fibrocytes, and a cytoplasm which is not basophilic at all, or but slightly basophilic and forms long branch-like protuberances interconnected by anastomoses (1). However, the presence of a great number of precollagenous fibres and chromotropic substances shows that the histogenesis of this tissue has not yet been accomplished. Most of fibroblasts of the connective tissue of the fin are known to retain their capacity to further development and metamorphoses, transforming into histioid forms in the case of a metamorphosis (2) and into cells of regeneration blasteme during regeneration (1, 3). On well spread total preparations of a regenerating extremity of a tadpole the network of fibroblasts near the wound surface may be seen to gradually transform into a network of regeneration elements («regeneration cells», according to Ide-Rozas). According to Ide-Rozas, these «regeneration cells» possess a large nucleus, a nucleolus clearly visible on the preparation, a cytoplasm endowed with basophilic properties: they are at the same time morphologically similar to embryonic cells. The basophilic properties of the cytoplasm of the regeneration cell (as well as any other cell) are due to the presence of pentose-nucleic acid compounds (4) which are capable of a peculiar coacervation in the shape of granules (crinome) (5, 7) during the reaction with vital neutral-red stains. This property of forming crinomes appears only at a definite developmental stage of the animal, and its absence is a sign that the cell is young (7). In the tadpole, for instance, this capacity is manifested prior to the disappearance of the exterior branchiae. At earlier stages, the cytoplasm of the cells of tadpoles kept in solutions of neutral-red is intensely basophilic, but devoid of granules (crinome).

The purpose of the present work was to find out whether the fibroblasts of the connective tissue of the fin actually transform into cells of the regeneration blasteme, and whether the nucleic exchange of the cells approaches that of the embryonic cells. As indices of such changes might serve variations in the amount of pentose-nucleic acid compounds in the cytoplasm of the cells, and their capacity of coacervation in the shape of granules in the case of reaction with neutral-red.

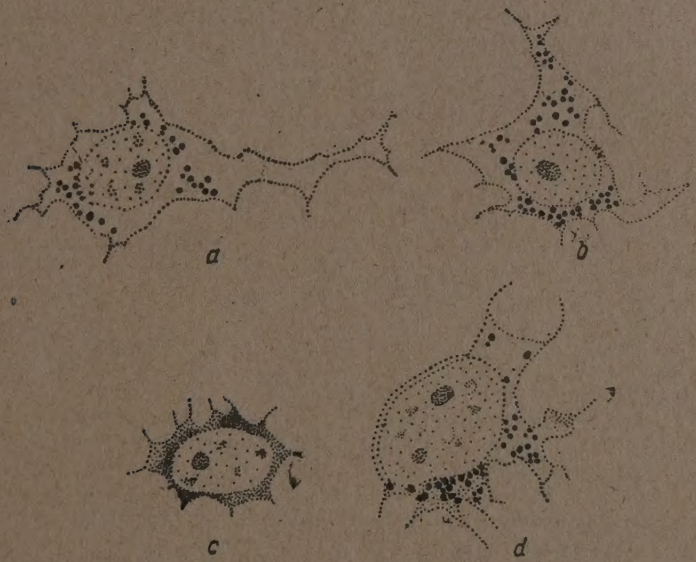
The following experimental method was applied. The caudal end was simultaneously amputated in several axolotls. After one or two days one of the axolotls was placed for 24 hours into a 1 : 100 000 solution of neutral-red. The axolotl stained dark-red. The caudal end bearing the regenerate was reamputated and fixed in Kedrovsky's modification of Birch-Hirschfeld's solution. The sections were stained eosin-azur. The experiment was run in two series on 28 axolotls.

On frontal sections made through the caudal fin, cells located immediately near the wound surface were examined. Only the fin part of the regenerate was studied. Attention was given to the size of the nucleus, the visibility of the nucleolus, the outlines of the cell body and the capacity of the pentose-nucleic acid compounds to coacervation in the shape of granules during the neutral-red reaction.

Three cases are usually met with in the case of vital staining of the living cell with neutral-red. In the first case all the pentose-nucleic acid compounds of the cytoplasm are found capable of coacervation in the shape of granules. In sections

stained eosin-azur, blue granules are visible against an utterly colourless background (complete segregation, after Kedrovsky). In the second case only some of the pentose-nucleic acid compounds are capable of coacervation, and blue granules are observed against a light-blue, clearly basophilic background (incomplete segregation, after Kedrovsky). In the third one the pentose-nucleic acid compounds are entirely incapable of coacervation in the shape of granules during reaction with neutral-red. The cytoplasm of these cells is entirely blue without any granules.

My observations showed that in the course of the regeneration process these three cases alternate in a definite regularity, corresponding to the regeneration stages. Ide-Rozas (1936) has described three such stages: 1. Stage of closing of wound surface with epithelium. The inflammation process in the subepithelial



Cells of axolotl regenerates: *a*, No. 23; *b*, No. 25; *c*, No. 28; *d*, No. 31

tissues has not begun. In the adult axolotl this stage may last for several days. 2. Inflammation stage. 3. Stage of tissue differentiation. The inflammation processes at this stage are usually achieved.

The regularity noticed by me consisted in the following: at the first regeneration stage the protoplasmic part of the cell retained its long protuberances. The pentose-nucleic acid compounds of the cytoplasm become entirely coacervate in the shape of granules (complete segregation of the crinome). The inset of the second (inflammation) stage of regeneration is characterized by the development of large cytoplasmic cells, by a reduced or totally lacking capacity of the pentose-nucleic acid compounds to segregation in the shape of granules; the cell has a basophilic plasma, and when stained with eosin-azur, takes on different tints of blue in various regions. With the beginning of tissue differentiation that follows the inflammation stage, resulting in the transformation of the «regeneration cells» of the regenerate's fin into fibroblasts (Ide-Rozas), the pentose-nucleic acid compounds of their cytoplasm become capable of complete coacervation in the shape of granules (crinome) (figure). It was out of the scope of this work to trace the reverse transformation of the «regeneration cells» into a typical fibroblast till the end of the process, because the time necessary for this examination would surpass the extreme fixation terms of the regenerate, viz. 31 days upon amputation. During this period the cell nuclei of the regeneration blasteme remained large and stained feebly with basic stains. Their nucleoli were distinctly visible (figure).

Below are given some of the most characteristic cases from the second series.

No. 23. Four days after amputation. Regeneration stage I. Nucleus—large and light. One or two clearly visible nucleoli. The cytoplasmic part has long branch-like protuberances. The blue granules of the crinome are on a colourless background (complete segregation).

No. 25. Nine days after amputation. Regeneration stage II. Large, light nucleus; one or two clearly discernible nucleoli. The cytoplasmic part shows shorter protuberances than in No. 23. In the fin region of the regenerate the cytoplasm of the cell shows blue granules against a colourless background (complete segregation). In the central region of the caudal regeneration surface, crinome is observable along with purely blue basophilic regions of plasma (incomplete segregation) and intensely basophilic cells without crinome.

No. 26. Eleven days after amputation. Regeneration stage II. Same kind of nucleus and nucleolus. Protoplasmic part with very short protuberances. Blue or light-blue plasm of different tints at various cell regions, mostly (with rare exceptions) without crinome.

No. 28. Fifteen days after amputation. Picture identical with No. 26.

No. 30. Nineteen days after amputation. Regeneration stage III. Large, light nucleus, clearly visible nucleoli. Two zones of the regeneration cone are visible, as pointed out by Ide-Rozas. According to this author, the inner dark zone of the regenerate consists of typical regeneration cells, while the outer light part consists of fibrocyte-like cells. In axolotl No. 30 the cells of the most distal regions of the regeneration cone contain a crinome against a basophilic background (incomplete segregation). Next comes a region, where the overwhelming majority of the cells devoid of crinome are filled with light-blue or blue plasm. All cells have short protuberances.

No. 31. Twenty one day after amputation. Regeneration stage III. Large light nucleus, clearly visible nucleoli. The plasmatic part shows short protuberances. Complete or almost complete segregation of the crinome.

In order to compare the cells of the regenerate with those of the axolotl embryo, vital staining of the latter with neutral-red was carried out beginning with the gastrula till the larval hatching stages. It has been found that at all stages, except the last one, the cell plasma when intensely basophilic, contains no crinome granules. At various developmental stages of the larva approaching hatching, the mesenchyme cells contain large light nuclei, large nucleoli clearly visible on the preparations, and a cell body with short protuberances and a basophilic plasm devoid of crinome. The similarity of these cells with those of the regeneration blasteme at the inflammation stage is indubitable. Only in a larva, fixed on the day of hatching, along with these cells one could observe cells with complete or incomplete segregation of the crinome. Thus, in the course of the regeneration process the cells of the connective tissue of an axolotl fin undergo the following changes. Their nucleus becomes large, feebly basophilic, the nucleolus becomes clearly visible, the protuberances of the cell body contract; the coacervation capacity of the cytoplasmic pentose-nucleic acid compounds in response to neutral-red is reduced, and, finally, it entirely disappears at the inflammation stage. At this moment the cells of the regeneration blasteme become similar to the embryonic cells at stages approaching the hatching of the larva not only morphologically, but by the state of the pentose-nucleic acid compounds and of the cytoplasm as well. Thereupon reverse changes set in, as a result of which the regeneration cell is transformed into a fibroblast again.

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30. V. 1943.

REFERENCES

[1] A. Ide-Rozas, Roux' Archiv, **135** (1936—1937). [2] А л е ш и н, Архив анат., гист. и эмбриол., **15**, вып. 1 (1936). [3] W. Hellmich, Roux' Archiv, **121** (1930). [4] I. Brachet, C. R. Soc. Biol., CXXXIII, No. 1 (1940). [5] N. G. Chlopin, Arch. exper. Zellf., **4**, 462 (1927). [6] T. Casperson and J. Schultz, Nature, **143**, 602 (1939). [7] Б. К е д р о в с к и й, Биологич. журн., IV, № 5—6 (1937).

Prix $ 0.50